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4TH INTERNATIONAL CONFERENCE: CONSTRUCTIVE MATHEMATICAL ANALYSIS

ABSTRACT BOOK

01-02 JULY 2026

The Fourth International Conference: Constructive Mathematical Analysis

Abstract Book

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TÜBİTAK

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FOREWORDS

Dear Participants,

It's my pleasure to chair The International Conference on "4th International Conference: Constructive Mathematical Analysis". 4th International Conference: Constructive Mathematical Analysis is an activity of the journal Constructive Mathematical Analysis. The first two series of the event were organized as workshops. Based on the requests from international researchers studying on constructive mathematical analysis, this series was organized as a conference. Our 2025 conference was successfully completed, and this 2026 edition is organized with the valuable support and partnership of Selçuk University (Türkiye), Gazi University (Türkiye), Lamberto Cesari Inter. Res. Center (Italy), Vasyl Stefanyk Carpathian National University (Ukraine), Thammasat University (Thailand), ANA&A (Italy), and The Scientific and Technological Research Council of Türkiye (TÜBİTAK). Specifically, this conference is supported by the Scientific and Technological Research Council of Türkiye (TÜBİTAK) 1001-Project 123F123.

This year, the conference proudly features a **"Special Session Dedicated to the 75th Birthday of Prof. Ioan Raşa"**. This session serves as a tribute to Prof. Raşa's outstanding achievements, invaluable contributions to mathematical science, and his inspiring academic legacy.

The main goal of this conference is to promote, encourage, and provide a forum for the academic exchange of ideas and recent research works on any field of Analysis and Function Theory. The conference presented new results and future challenges, in a series of keynote lectures and contributed short talks. We thank invited speakers distinguished **Prof. Carlo Bardaro**, distinguished **Prof. Raul Curto** and distinguished **Prof. Feng Dai** for contribution to the our conference.

The full texts of the papers presented in this conference will be published in *Springer Proceedings in Mathematics & Statistics*. The papers presented in this conference will be considered in the special issues of journals *ALTAY Conference Proceedings in Mathematics, Applied Mathematics, Carpathian Mathematical Publications, Constructive Mathematical Analysis, Gazi University Journal of Science, Mathematical and Computational Applications, Journal of Mathematical Sciences* and *Symmetry*.

This booklet contains the titles and abstracts of all invited and contributed talks at the *The Fourth International Conference: Constructive Mathematical Analysis* and is available on the conference website.

Prof. Dr. Tuncer ACAR
On Behalf of Organizing Committee - Chairman

We warmly congratulate Prof. Ioan Raşa on his 75th birthday and wish him a healthy, happy and long life.

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Contents

4th International Conference: Constructive Mathematical Analysis

Exponential Sampling Theory: Error Estimates via Mellin Distance

Carlo Bardaro

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key-words: Mellin transform, Mellin distance, exponential sampling formula.

Abstract:
In the framework of Mellin–Lebesgue spaces, we introduce (see [1] and [2, Chapter 10]) a new notion of distance, using the inversion formula of the Mellin transform. This notion represents an efficient tool for estimating errors for some approximate basic formulae of Mellin analysis. In particular, we study the approximate versions of the exponential sampling formula, the reproducing kernel formula in Mellin setting, the Bernstein inequality for not necessarily band-limited functions in Mellin sense. It is proved that these estimates are asymptotically best possible and can be used also for obtaining characterizations of functional spaces in terms of the speed of convergence to zero of the errors. We study error estimates when the function f belongs to Mellin–Paley–Wiener spaces, Mellin inversion classes, Mellin–Lipschitz classes, Mellin–Sobolev spaces and Mellin–Hardy spaces, so obtaining a sort of hierarchy of these spaces in terms of the speed of convergence to 0 of the error.

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4th International Conference: Constructive Mathematical Analysis

Classes of Operators Related to Subnormal Operators

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key-words:

Abstract:
In this talk we lay the foundations for a new theory encompassing two natural extensions of the class of subnormal operators, namely the n–subnormal operators and the sub-n–normal operators, where $n \in \mathbb{N}$. We discuss inclusion relations among the above-mentioned classes and other related classes, e.g., n–quasinormal and quasi-n–normal operators. We show that sub-n–normality is stronger than n–subnormality, and produce a concrete example of a 3–subnormal operator which is not sub-2–normal. In [1], R.E. Curto, S.H. Lee and J. Yoon proved that if an operator T is subnormal, left-invertible, and such that T^2 is quasinormal, then T is quasinormal. In subsequent work, [3], P.Pietrzycki and J. Stochel improved this result by removing the assumption of left invertibility. In this talk we will consider suitable analogs of this result for the case of operators in the above-mentioned classes. In particular, we prove that the weight sequence of an n–quasinormal unilateral weighted shift must be periodic with period at most n.
The talk is based on joint work with Thankarajan Prasad, Nalanda University, India, which has recently appeared in print [2].
Research partially funded by U.S. National Science Foundation DMS-2247167.

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Sampling Discretization of Integral Norms in Finite-Dimensional Spaces

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key-words: Marcinkiewicz discretization, random sampling, empirical L_p moments.

Abstract:
In this talk, I will present recent advancements in the Marcinkiewicz discretization problem using random sampling in finite-dimensional Spaces. The goal is to establish two-sided estimates for the integral norm of functions in the space via a finite sum of function values evaluated at randomly selected points that are independent of the individual functions in the space. The main challenge is to determine the "nearly" optimal number of random points required for the Marcinkiewicz discretization inequalities to hold with high probability.

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Investigation of Physics-Informed Neural Networks for Solving the Allen-Cahn Equation

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key-words: Physics-informed neural networks, Allen-Cahn equation, scalar auxiliary variable.

Abstract:
Physics-Informed Neural Networks (PINNs) offer a powerful deep learning framework for solving nonlinear partial differential equations. However, standard PINNs face significant challenges when applied to the Allen-Cahn equation, particularly concerning sharp interface resolution, spectral bias, and optimization stability during long-time integration. In this paper, we propose a novel Thermodynamically-Consistent Spectral-PINN architecture. To overcome optimization failures and Gibbs phenomena associated with classical spectral methods near sharp interfaces, we utilize Fourier Feature Positional Encoding. Furthermore, to guarantee strict energy dissipation without exacerbating the non-convexity of the loss landscape, we introduce a Scalar Auxiliary Variable (SAV) formulation into the PINN framework. We provide rigorous theoretical justification for the proposed architecture, including a proof that the modified energy functional is monotonically decreasing along solutions of the SAV-reformulated system. Preliminary numerical experiments are currently underway to validate the theoretical predictions.

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Modified Process Capability Index under Unit-Gumbel Distribution: Comparative Analysis of Point Estimators

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key-words: Process capability index, unit distribution, Monte Carlo simulation.

Abstract:
Process capability indices are fundamental statistical indicators that quantitatively measure a process's capacity to produce output within defined specification limits and are of strategic importance in quality control processes. This study presents a specialized C_{GPK} index based on the unit Gumbel distribution to determine the process capability index for unit distributions, a limited area in the literature. In this study, point estimates are obtained for the C_{GPK} index using different statistical methods. To determine the effectiveness of the obtained estimators, the performance measures used bias, mean squared errors, and mean relative errors are compared through a comprehensive simulation study. The simulation results reveal the effectiveness of the estimation methods under different sample sizes and parameter combinations. Theoretical findings, applications conducted on various datasets defined within the unit range (0,1), and the contribution of this index to the literature and its practical advantages in process performance evaluation have been proven.

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Geometric Analysis of Magnetic Trajectories of Charged Particles and Their Relations with Associated Curves

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key-words: Magnetic Trajectory, associated curves, Lorentz equation.

Abstract:
This paper focuses on the geometric analysis of trajectories of charged particles moving under the influence of magnetic fields. The primary aim is to reveal the relationships between these trajectories and special curves in differential geometry. In this context, magnetic curves are examined in relation to well-known curve pairs, and several of their fundamental geometric properties are established. Finally, illustrative examples are presented to support the theoretical results.

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OMNIAI: AI-Powered Chatbot-Based Appointment Management System

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key-words: Artificial intelligence, cloud computing, microservices.

Abstract:
The rapid digitalization of customer-business interactions has highlighted the inefficiencies of traditional, manual appointment management systems, which are often prone to high latency, human error, and limited operational hours. To resolve these challenges, this study presents OmniAI, an artificial intelligence-powered, chatbot-based appointment management system designed to fully automate the reservation process and optimize administrative workflows. The proposed method leverages a microservices architecture, predominantly developed using the Go programming language, to ensure high scalability and modularity. The system interfaces directly with end-users via the WhatsApp messaging platform using the Twilio API. To handle concurrent requests and maintain high performance under heavy traffic, asynchronous message queuing is implemented via RabbitMQ. The core intelligence of OmniAI is driven by an AI Service integrated with OpenAI's GPT-4o model. This is significantly enhanced by a Retrieval-Augmented Generation (RAG) module, which dynamically queries a PostgreSQL-backed corporate knowledge base. This integration enables the chatbot to fetch context-specific data—such as working hours, branch details, and service descriptions—facilitating highly accurate intent analysis and context-aware natural language responses. Furthermore, the system includes a comprehensive frontend dashboard built with React and TypeScript, allowing business administrators to monitor daily reservations, manage user roles with JWT-based authentication, and track AI performance analytics in real-time. The entire ecosystem is containerized using Docker and securely routed through a Traefik API gateway, ensuring isolated and portable deployments. The implementation results demonstrate that OmniAI substantially decreases average response times and reservation errors compared to traditional methods, while maintaining a high rate of successful automated bookings. In conclusion, the integration of advanced Large Language Models with RAG in a microservices framework provides a robust, 24/7 automated booking experience, significantly reducing administrative overhead and improving overall customer satisfaction.

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On Integral Curves Generated by PAF in Euclidean 3-Space

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key-words: Integral curves, associated curves, Euclidean space.

Abstract:
This study, we introduces a family of integral curves associated with a curve according to PAF in Euclidean 3-space.The curvature relations among these curves are analyzed comprehensively. Moreover, the developed approach offers constructive procedures for obtaining certain special curves. To illustrate and validate the theoretical findings, several representative examples are included.

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4th International Conference: Constructive Mathematical Analysis

Fixed Point Approximation Schemes for Wasserstein Gradient Flows in Generative Modeling

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Keywords: Wasserstein gradient flow, generative modeling, proximal operator.

Abstract:
We study a conditional fixed point framework for a particle-based Wasserstein proximal sampler. Motivated by the Jordan–Kinderlehrer–Otto scheme, we regard a Wasserstein proximal step as a self-map and define Picard, Mann and Ishikawa updates through selected Wasserstein geodesics. Under explicit contractivity and selected-geodesic metric-convexity assumptions, we prove convergence and perturbation estimates. In addition, we identify the empirical quadratic proximal step induced by optimal matching, which provides a controlled common-coupling particle surrogate where the assumptions can be checked. Numerical experiments on two-dimensional targets compare fixed-target, mini-batch, and resampled-target regimes. Picard/JKO is the strongest method in deterministic fixed-target tests, while relaxation, especially Mann relaxation, can reduce the final empirical W_2 error when the target batch is resampled. The results are intended as a stability analysis for this particle surrogate, not as a global contractivity theorem for general Wasserstein proximal maps or as a large-scale generative modeling claim.

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4th International Conference: Constructive Mathematical Analysis

Two Dimensional Fuzzy Differential Transform Method for Solving of Two Dimensional Nonlinear Fuzzy Volterra Integral Equations

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key-words: Fuzzy differential transform method, two dimensional fuzzy Volterra integral equations, fuzzy calculus.

Abstract:
This study applies the two-dimensional fuzzy differential transform method (2D-FDTM) to solve twodimensional fuzzy Volterra integral equations (2D-FVIEs). The fundamental properties of the method are established, and illustrative examples are provided to verify its efficiency and accuracy. The results confirm that the proposed approach offers a reliable analytical tool for handling fuzzy Volterra integral equations in two dimensions. .

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On the Approximation of Borel Derivatives by Nonlinear Szász Mirakyan Durrmeyer Operators

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key-words: Borel derivative, nonlinear operator, pointwise convergence.

Abstract:
In the present study, we consider a sequence of nonlinear Szász-Mirakyan-Durrmeyer operators (NS_n) of the form

$$(NS_n)(f;x) = \int_0^\infty K_n(x,t,f(t))\,dt \ , \ x \in [0,\infty) \ , \ n \in \mathbb{N},$$

acting on bounded functions on every finite subinterval of $[0,\infty)$ where $K_n(x,t,u)$ satisfy some suitable assumptions.

We give some approximation results with regard to the convergence of the operators NS_n to right, left, and symmetric Borel differentiable functions.

Our study extend some of the previous results about the linear integral operators and also aims to bring a new perspective to research in approximation with nonlinear operators by introducing a different concept of a derivative.

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Almost Automorphic and Pseudo-Almost Automorphic Solutions: A Unified Theory with Applications to Classical and Fractional Evolution Systems

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key-words: Almost automorphic functions, pseudo almost automorphic functions, parabolic equations.

Abstract:
This paper studies almost automorphic and pseudo-almost automorphic mild solutions for a class of nonlinear evolution equations in Banach spaces, covering both classical and fractional frameworks. A unified approach is developed based on semigroup and resolvent theory, together with fixed-point techniques, to establish existence and uniqueness results for semilinear equations generated by exponentially stable $\mathbb{C}_0$–semigroups and their fractional counterparts. The novelty lies in providing a coherent framework that simultaneously treats classical and fractional dynamics while incorporating both almost automorphic and pseudo-almost automorphic structures. The results extend several known contributions in the literature. Applications are presented to parabolic integrodifferential equations with memory, the Schrödinger–Poisson system, and classical and fractional Navier–Stokes equations, including a concrete example illustrating the pseudo-almost automorphic behavior of the forcing term.

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Numerical Study on a Class of Higher-Order Singular Polytropic Equations

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key-words: Collocation method, fractional Legendre polynomials, operational matrices.

Abstract:
In this work, we address the numerical approximation of a class of higher-order singular polytropic equations (SPEs) with variable coefficients, written in the general form [2]

$$x^{-\eta}\, D^{\beta}(x^{\eta}\, D^{\gamma} v(x)) = \omega(x, v(x)), \tag{1}$$

where $v(x)$ is the unknown function, $\eta > 0$ is a real shape parameter, $\omega(x, v(x))$ is a given (possibly nonlinear) source term, and β, γ are Caputo fractional differentiation orders, with $\beta + \gamma$ defining the total order of (1).

We develop a collocation scheme based on shifted fractional-order Legendre polynomials $\mathcal{L}_n^{(\alpha)}(x)$ on $[0, 1]$. Operational matrices for fractional integration and product are constructed in this basis, reducing (1) to a linear or nonlinear algebraic system for the spectral coefficients. A rigorous error bound confirms the reliability and convergence of the proposed approach. Numerical experiments on fourth- and fifth-order test problems show that an appropriate choice of α reduces the absolute error by several orders of magnitude while requiring fewer basis functions than existing techniques such as the Haar wavelet method [1].

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On a New Modification of the Mellin-Gauss-Weierstrass Operators in the Weighted Mellin-Lebesgue Spaces

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key-words: Logarithmic weighted modulus of continuity, logarithmic moments, weighted Mellin-Lebesgue spaces.

Abstract:
In this presentation, we establish a quantitative convergence result for a new generalization of Mellin-Gauss-Weierstrass operators preserving logarithmic functions, using the logarithmic weighted modulus of continuity. We derive the logarithmic moments of the modified operators and express a Voronovskaya-type theorem. Furthermore, we obtain the rate of convergence and show global smoothness preservation property in weighted Mellin-Lebesgue spaces of Lebesgue measurable functions.

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On Some Matrix Transformations of Sequences of Function

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key-words: Cesàro matrix, function sequences, Zweier matrix.

Abstract:
In mathematical analysis, the convergence of function sequences and series is a fundamental concept in both theoretial and applied mathematics. Different types of convergence of function sequences and their behavior in function spaces are significant area of research analysis and functional analysis. In recent years, research has focused on alpha convergence, one of the important types of convergence of function sequences, as defined below, and its generalizations. Let (X,d) be a metric space. Let $(f_n) \in \mathfrak{sf}(\mathbb{R}^X)$, $f \in \mathbb{R}^X$, and $x_0 \in X$ be given. The sequence of functions (f_n) is said to be α-convergent to the function f at the point x_0 if $f_n(x_n) \to f(x_0)$ for every sequence $(x_n) \subset X$ such that $x_n \to x_0$ [1].

In this study, we investigate the convergence properties of real-valued function sequences defined on a metric space that are pointwise convergent, uniformly convergent, alpha-convergent, and uniformly alpha-convergent under Zweier and Cesàro matrix transformations. We will also examine the behavior of sequences with pointwise and uniform exhaustiveness properties under these summability matrices.

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A Neurodynamical Neural Network Approach for Solving Support Vector Machine Problems with Generalized Pinball Loss Functions via Hyperbolic Tangent Smoothing Technique

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key-words: Neural networks, support vector machines.

Abstract:
In this paper, we propose a neurodynamical neural network approach for solving support vector machine problems with generalized pinball loss function. First, we use the hyperbolic tangent smoothing technique to find a smooth approximation of the generalized pinball loss associated with the SVM problem. Then we propose a neurodynamical neural network model to solve the given support vector machine problem with smooth objective function. We prove the equivalence between the set of optimal solutions the smoothed optimization problem and the set of equilibrium points of the proposed model. Then we analyze the existence and uniqueness of solutions, stability, and convergence of the method. Next we conduct numerical simulations both on synthetic and UCI datasets to show the effectiveness of the method. We evaluate the performance of the model using the accuracy, Matthews Correlation Coefficient (MCC), and F1-score metrics. Finally, we compare our model against existing models of solving support vector machine problems with pinball loss functions. We conduct the Friedman test statistical analysis followed by the Nemenyi post-hoc analysis to examine the statistical difference of the proposed model against the existing methods.

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Where/Why Do We Use the Bivariate/Multivariate Positive Linear/Nonlinear Operators?

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key-words: Positive linear/nonlinear operators, computer-aided geometric design, neural networks.

Abstract:
Approximation theory is a fundamental branch of mathematical analysis concerned with the representation of complex functions by simpler, more computationally tractable ones, such as polynomials, splines, or rational functions. In this presentation, we investigate the theoretical motivations and practical implementations of bivariate and multivariate positive linear and nonlinear operators within the framework of approximation theory.
Acknowledgements: The author has been supported by the Scientific and Technological Research Council of Turkey (TUBITAK)/ 2224-B-Grant Program for Participation in Scientific Meetings within the Country.

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Existence of Solutions for a Fractional $p(x,\cdot)$-Laplacian Problem with Neumann Boundary Conditions on an Exterior Domain

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key-words: Fractional $p(x,\cdot)$-laplacian, Nehari manifold, nonlocal nonlinear boundary condition, exterior domain.

Abstract:
This paper addresses the existence of non-negative solutions for a fractional $p(x,\cdot)$-Laplacian problem with nonlocal nonlinear boundary conditions on an exterior domain. The study is conducted using the Nehari manifold approach, which is the primary instrument for establishing the key results.

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Magic Squares: From Ancient Aesthetics to a New Recursive Construction Method

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key-words: Magic squares, recursive construction, matrix theory.

Abstract:
The history of solving quadratic equations stretches back over four thousand years to Ancient Babylon, where scribes calculated land areas and astronomical cycles using rules remarkably similar to our own. However, most cuneiform texts provide only "recipes"—step-by-step instructions without explaining the underlying logic. Paradoxically, modern mathematics education often mirrors this ancient approach, favoring universal formulas (the "standard recipe") while neglecting more elegant and efficient alternatives. This study demonstrates that a wide range of problems typically solved via standard quadratic equations can be addressed more effectively through the direct application of the difference of squares formula.

We propose a specific algorithm based on this approach, highlighting its relevance for mathematical modeling. Many real-world problems, when initially translated into mathematical language, naturally take a form that is ideally suited for the difference of squares method before they are forcibly reduced to a standard quadratic form. By revisiting techniques once championed by Diophantus, Al-Khwarizmi, and Fibonacci—methods largely overshadowed since the 17th century—this work offers a streamlined alternative to the "general recipe," enhancing both computational efficiency and conceptual clarity in the classroom.

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Eigenvalue Expansions for a Conformable Sturm–Liouville Operator with Mixed Boundary Conditions

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key-words: Conformable derivative, Sturm–Liouville operator, perturbation theory.

Abstract:
In this paper, we study the conformable Sturm–Liouville problem

$$-D^{2\alpha}y(x) = \lambda y(x), \qquad y(0) = 0, \quad D^{\alpha}y(1) = 0, \tag{1}$$

on the unit interval, where $D^{\alpha}y(x) = x^{1-\alpha}y'(x)$ denotes the conformable derivative [2]. A Dirac delta perturbation of the form $V(x) = \delta(x - a)$ is introduced, and the corresponding first- and second-order eigenvalue corrections are derived. We analyze the critical points of the first-order correction with respect to the perturbation point a and the conformable order α. As the two endpoints are not symmetric, we study the behaviour of the spectrum under perturbations near the Neumann end $x = 1$. Moreover, the cumulative response over a right subinterval is obtained in closed form.

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Appell-Vietoris Polynomials and Their Zeros

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key-words: Vietoris number sequence, Appell-Vietoris polynomials, zero bounds.

Abstract:
The Vietoris numbers form a sequence of pairwise repeated rational numbers that originally arose in the study of the positivity of certain trigonometric sums (cf. [2]). In this talk, we consider the class of Appell polynomials associated with the Vietoris sequence, introduced in [1], and investigate several of their structural properties. In particular, we derive lower and upper bounds for their zeros in the complex plane.

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Approximation Processes on Minimal Surfaces

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key-words: Minimal surfaces, non-linear Bernstein-Schnabl operators, Voronovskaja formula.

Abstract:
We point out the non-additive potential structures and continuous-time evolutionary processes generated by a class of non-linear Bernstein-Schnabl type operators on minimal surfaces. Through a generalized Voronovskaja-type formula, we compute the uniform limit and the connection with a degenerate second-order elliptic differential operator whose coefficients are explicitly governed by the intrinsic geometry. For a fixed initial datum f, the evolution trajectory remains strictly constrained to the Riemannian metric g_f spanned by that datum. This allows us to treat the non-linear iteration identically to a sequence of positive linear contractions. This extends to a much wider context the results in [1, 2].

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Statistical Inference of a New Four-Parameter Marshall Olkin Distribution: Comprehensive Simulation and Measurement Applications

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key-words: Marshall Olkin family, Monte Carlo simulation, measurement datasets.

Abstract:
In this study, a new flexible four-parameter Marshall–Olkin Fréchet Power Rayleigh (MOFPR) distribution is introduced, which is constructed by applying the Marshall–Olkin [2] family of distributions to the Fréchet Power Rayleigh [1] model, and its statistical properties are examined. The fundamental mathematical and statistical characteristics of the proposed model, including the probability density function, cumulative distribution function, hazard rate function, and quantile function, are derived. To illustrate the model's versatility, the behavior of the probability density function and cumulative distribution function are analyzed through graphical representations under various parameter configurations. A significant contribution of this research is the comprehensive estimation framework, where twelve different estimation methods—including maximum likelihood, least squares, weighted least squares, Anderson–Darling, and Cramér-von Mises, among others—are employed to estimate the unknown parameters. The finite-sample performance of these estimators is assessed through an extensive Monte Carlo simulation study involving 5,000 replications. The efficiency of the estimators is compared based on several performance metrics, such as bias, mean squared error, mean relative error, and average absolute bias. Furthermore, the practical utility of the proposed distribution is demonstrated using two real measurement datasets. The modeling capability of the new distribution is evaluated via goodness-of-fit statistics, including Kolmogorov-Smirnov, Anderson-Darling, and Cramér-von Mises tests. Comparative results indicate that the proposed model provides a superior fit over several distributions in the literature, such as the Weibull, gamma, exponential, Fréchet, and Rayleigh.

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Inference for the Reliability and Mean Time to Failure of Protected Consecutive k-out-of-n: G Systems

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key-words: Consecutive k-out-of-n: G system, protection block, reliability.

Abstract:
This study develops inference for protected consecutive k-out-of-n:G systems under a complete lifetime sampling scheme. The known-parameter reliability structure is rewritten in terms of compact elementary blocks, which provide a convenient basis for estimation when the failure rates are unknown. By integrating these reliability blocks, compact closed-form expressions are obtained for the corresponding mean time to failure quantities. Using the resulting block representations, plug-in maximum likelihood estimators are constructed for the reliability and mean time to failure measures. The finite-sample behavior of these estimators is investigated by deriving their expected values and variances. An illustrative complete-data example demonstrates the implementation of the proposed estimators together with parametric bootstrap confidence intervals.

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A Study on Associated Curves and Their Curvature Relations

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key-words: Associated curves, direction curves, Euclidean space.

Abstract:
This study, we introduces a class of associated curves constructed as integral trajectories of vector fields defined by the Positional Adapted Frame in Euclidean 3-space. The relationships between the curvature functions of these curves are examined in detail. Furthermore, the presented method provides constructive techniques for generating some special curves. Several illustrative examples are also presented to support the theoretical results.

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Stability to Perturbations of Modified S-Fractions with Applications

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key-words: Modified S-fraction, stability to perturbations, set of uniform stability, relative error.

Abstract:
This presentation is devoted to the study of stability to perturbations for modified S-fractions (modified Stieltjes continued fractions) of the form

$$\cfrac{a_0}{z+\cfrac{a_1}{z+\cfrac{a_2}{z+\ddots}}}, \qquad a_k > 0, \quad k \geq 0, \quad z \in \mathbb{C}. \tag{1}$$

While this class of functional continued fractions is a powerful tool in applied problems [1], its practical application requires a thorough understanding of the stability to perturbations [2].

We establish a formula quantifying the relative error of the approximants resulting from perturbations applied to the coefficients a_k of the modified S-fraction (1) itself. Utilizing this formula and leveraging techniques from the analytical theory of continued fractions, we derive sufficient conditions for the stability of (1) to perturbations.

We distinguish between two notions: the set of pointwise stability and the set of uniform stability. Specifically, assuming

$$0 < a_k \leq A, \quad 0 < \hat{a}_k \leq A, \quad k \geq 0,$$

where $\hat{a}_k$ are the perturbed coefficients, we prove that the set

$$\Omega_A = \{z \in \mathbb{C} : |\operatorname{Re} z| > \sqrt{A(1 - |\operatorname{Re} z|/|z|)}\}$$

is a set of stability to perturbations, and the set

$$\Omega_{\alpha,C} = \{z \in \mathbb{C} : |\operatorname{Im} z| \leq |\operatorname{Re} z| \tan\alpha,\ |\operatorname{Re} z| \geq C\}, \quad 0 \leq \alpha < \pi/2, \quad C > \sqrt{A(1-\cos\alpha)},$$

is a set of uniform stability to perturbations of the modified S-fraction (1).

A significant outcome of this research is the derivation of explicit bounds for the relative errors of the approximants, which demonstrate how the perturbation error of a continued fraction's approximant depends on the magnitude of the perturbations and the values of the fraction's coefficients.

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A Chemotaxis-Navier-Stokes System with Dynamical Boundary Conditions

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key-words: Dynamical boundary conditions, chemotaxis, Navier-Stokes.

Abstract:
A chemotaxis-Navier-Stokes system is studied under dynamical boundary conditions in a bounded convex domain with smooth boundary. This models the interaction of populations of swimming bacteria with the surrounding fluid. The existence of a global weak solution is proved using multiple layers of approximations and Rothe's method for the time discretization.

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Korovkin-Type Approximation for Non-Positive Operators

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key-words: Korovkin-type theorems, non-positive operators, operator version, Grünwald operators.

Abstract:
The classical Korovkin theorem traditionally relies on the positivity of the underlying sequence of operators. However, in 1968, D. E. Wulbert established the first non-positive version. In this talk, I will present a generalized form of Wulbert's result to the class of uniformly bounded sequence of operators. As an application, I will discuss an operator version of this Korovkin-type theorem which will cover existing results in this direction. I will also present illustrative examples, which has its roots in the Grünwald's interpolation operator.

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A Generalized Fourier-Feynman Transform over Paths in Abstract Wiener Space

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key-words: Abstract Wiener space, analogue of Wiener measure, Fourier-Feynman transform, rotation theorem.

Abstract:
Let $(\mathbb{B}, \mathcal{B}(\mathbb{B}), \mu)$ be an abstract Wiener space and φ_a be a finite positive measure on $\mathcal{B}(\mathbb{B})$. Let $C^{\mathbb{B}}[a,b]$ denote an analogue of Wiener space over paths in abstract Wiener space, the space of $\mathbb{B}$-valued continuous functions on $[a,b]$ and let $w^{\rho,\beta,\mathbb{B}}_{a,b;\varphi_a}$ be a generalized analogue of Wiener measure over $C^{\mathbb{B}}[a,b]$ according to β, φ_a and ρ, where $\rho > 0$ and β is a strictly increasing function on $[a,b]$. In this talk, we provide a rotation theorem on the product space $(C^{\mathbb{B}}[a,b])^2$ given by

$$\begin{aligned}&\int_{(C^{\mathbb{B}}[a,b])^2} F(\cos\theta y_1 - \sin\theta y_2, \sin\theta y_1 + \cos\theta y_2) d(w^{\rho,\beta,\mathbb{B}}_{a,b;\varphi_a})^2(y_1,y_2)\\ = &\int_{(C^{\mathbb{B}}[a,b])^2} F(y_1,y_2) d(w^{\rho,\beta,\mathbb{B}}_{a,b;\varphi_a})^2(y_1,y_2).\end{aligned}$$

where F is any (Borel) measurable function on $(C^{\mathbb{B}}[a,b])^2$. We then establish several results related to the equation just stated. This rotation theorem generalizes Bearman's rotation theorem and is essential to define the Fourier-Feynman transform on $C^{\mathbb{B}}[a,b]$ which plays a significant role in the study of Feynman integration theory, especially, in the study of probabilistic approach to Brownian motion.

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A Neurodynamic Optimization Model for Smooth Pinball Loss Based Support Vector Machines

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key-words: Pinball loss, smooth function, neurodynamic optimization.

Abstract:
Pinball loss function is widely used in Support Vector Machine (SVM) models due to its robustness; however, its non-smooth property limits the applicability of gradient-based optimization methods. In this research, a smooth approximation of pinball loss is developed and incorporated into the SVM framework to improve differentiability and optimization efficiency. Furthermore, a neurodynamic-based optimization approach is proposed to obtain the optimal solution through a stable dynamic system under Karush–Kuhn–Tucker (KKT) conditions. The proposed method is theoretically analyzed and experimentally evaluated using benchmark datasets from the UCI benchmark dataset. Numerical results demonstrate that the proposed model achieves superior convergence behavior and classification performance compared with existing state-of-the-art methods.

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Automatic Differentiation Using Neural Networks and Their Comparison with Higher Order Spectral Differentiation

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key-words: Automatic differentiation, back propagation, spectral differentiation.

Abstract:
Automatic differentiation (AD), also called algorithmic differentiation, applies the chain rule from calculus step by step to compute the derivative while the program is running. That is why it performs faster and more accurately compared to other differentiations and is also easy to use. Calculate the derivative value and the function value simultaneously. AD is widely used in Physics-Informed Neural Networks (PINNs) and has important applications in computational fluid dynamics, atmospheric sciences, and engineering design optimisation. In this paper, we consider two continuous and two discontinuous functions. We will apply both the spectral derivative and AD methods to those functions and compare the results graphically. The spectral derivative matches very closely with AD for smooth functions. AD results are much better compared to the classical method, which shows oscillations near the boundary and at discontinuous points.

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4th International Conference: Constructive Mathematical Analysis

Fixed-Point Iterations for Analyzing the Topology of Biological Networks

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key-words: Fixed point theory, mathematical modeling, network topology.

Abstract:
Cellular decisions and metabolic balances in living systems are not the product of a single molecule, but rather emerge from the collective, rhythmic, and nonlinear interactions of numerous chemical components. Biological networks model this complexity as directed graphs $G = (V, E)$, where vertices represent intracellular components such as proteins, enzymes and metabolites, and edges represent activation or inhibition relationships between them. The insulin resistance signaling pathway in Type-2 Diabetes constitutes a concrete example of such a structure: a high-dimensional, nonlinear regulatory network governed by nested positive and negative feedback loops.

The dynamic behavior of this system is modeled by a system of nonlinear ordinary differential equations (ODEs) of the form

$$\frac{dx_i}{dt} = f_i(x_1, x_2, \ldots, x_n) - g_i(x_i), \quad i = 1, \ldots, n,$$

where $x_i(t) \in \mathbb{R}_{\geq 0}$ denotes the concentration of intracellular molecular components, f_i is the nonlinear Hill-type production function, and g_i represents the degradation rate. The fixed points of this system are the equilibrium positions $\mathbf{x}^* = (x_1^*, x_2^*, \ldots, x_n^*)^T$ satisfying $\frac{dx_i}{dt} = 0$. A stable fixed point symbolizes homeostasis, that is, a healthy cellular equilibrium, whereas an unstable fixed point or a bifurcation corresponds to the onset of disease pathogenesis. However, analytically determining these fixed points in high-dimensional systems is highly challenging due to the complexity of the Jacobian matrix $J \in \mathbb{R}^{n \times n}$.

To overcome this difficulty, we apply the Circuit-Breaking Algorithm (CBA) proposed by Radde (2010, 2012), which bridges dynamical system theory and graph theory. In this study, we modularly adapt this topology-based and parameter-independent framework to insulin signaling pathways and glucose-insulin regulation mechanisms to analytically identify the critical bifurcation thresholds triggering pathogenesis, thereby providing a scalable tool for the analysis of complex human disease networks [1, 2, 3].

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New Results on Fixed Point Theory in Perturbed Metric Spaces with Directed Graphs

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key-words: Perturbed metric spaces, fixed point theory, directed graphs.

Abstract:
This paper establishes new fixed point results for Geraghty-type contractive mappings on complete perturbed metric spaces endowed with directed graph structures. By integrating the concepts of perturbed metrics and graph-preserving mappings, a generalized Banach–Geraghty fixed point theorem is obtained, ensuring the existence and uniqueness of fixed points under suitable contractive conditions. The proposed framework simultaneously captures perturbation effects in distance measurements and structural relationships represented by directed graphs, thereby extending several classical fixed point results in metric and graph-based settings. Furthermore, the applicability of the theoretical findings is demonstrated through a nonlinear Fredholm integral equation, illustrating the effectiveness of the developed approach in the analysis of nonlinear problems involving uncertainty and interconnected structures.

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On the Approximations of the Ratios of Double Hypergeometric Functions by Branched Continued Fractions

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key-words: Double hypergeometric functions, branched continued fractions, approximation by rational functions, convergence, numerical stability.

Abstract:
The problems of approximating the ratios of double hypergeometric series by branched continued fractions are considered: construction of the simplest fractional structure and investigation of the convergence and numerical stability of the constructed expansion. New methods are proposed for investigating the convergence and establishing the sets of numerical stability of the expansions of the ratios of some double hypergeometric series, including the well-known Appell hypergeometric series, into branched continued fractions [1].

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Solution Structure of Higher-Order Nonlinear Hadamard-Type Fractional p-Laplacian Problems on Finite Domains

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key-words: Hadamard fractional derivative, nonlinear p-laplacian, fixed point methods.

Abstract:
We investigate a class of higher-order nonlinear boundary value problems involving Hadamard-type fractional operators and the p-Laplacian on a finite interval subject to integral boundary conditions. The problem is reformulated as an equivalent integral equation and analyzed within an operator-theoretic framework in a suitable Banach space.

Two independent solvability results are established. The first guarantees existence and uniqueness of solutions under standard continuity and growth assumptions via a contraction principle. Unlike earlier studies relying solely on Banach contraction arguments, the present analysis establishes existence results for fully nonlinear configurations under weaker hypotheses. The second result therefore provides the existence of at least one solution under more general nonlinear structures through alternative operator estimates.

This framework enables the treatment of fully nonlinear higher-order problems together with integral-type boundary conditions, extending several earlier results restricted to semilinear or lower-order settings. An illustrative example is provided to demonstrate the applicability of the theoretical findings. The obtained results contribute to the constructive theory of Hadamard-type fractional differential equations and provide a rigorous analytical foundation for nonlinear fractional p-Laplacian models.

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XBilal Distribution: Properties, Parameter Estimation and Applications

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key-words: Estimation, finite mixture distribution, Monte Carlo simulation.

Abstract:
This study presents the XBilal distribution, which is obtained as a special finite mixture of exponential and Bilal distributions. The fundamental mathematical and statistical properties of this new distribution, such as the probability density, moment generating, survival, and hazard function, have been derived. Different point estimators, including the maximum likelihood, least squares, weighted least squares, Cram´er-von Mises, Anderson-Darling, and maximum product of spacings have been considered for estimating the model parameters. To empirically support the theoretical findings of the proposed distribution, a comprehensive Monte Carlo simulation study is conducted. This simulation process is implemented under different sample sizes and parameter combinations, comparing the performance of the estimators considered. The efficiency, consistency, and relative superiority of the estimators are analyzed using statistical performance criteria such as bias and mean squared errors. Finally, an application study is carried out on a real dataset to demonstrate the flexibility of the proposed distribution and its superiority over traditional models. The findings show that the XBilal distribution provides a better fit than existing models and is an effective alternative in data modeling.

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On a Statistical Versions of Certain Types of Alpha Convergence

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key-words: Alpha convergence, function sequences, statistical convergence.

Abstract:
This study considers new types of alpha type convergences in the context of statistical convergence for sequences of functions defined between metric spaces. Furthermore, using this concepts, we prove some fundamental properties and get connections between them.

Let (X,d) and (Y,ρ) be metric spaces, (f_n) be a sequence of functions and f be a function from X to Y.

Definition 1. [1] The function sequence (f_n) is called *uniform alpha convergent* to the function f on X if for every sequence (x_n) and (y_n) of points of X with the property $d(x_n,y_n)\to 0$, the sequence $\rho(f_n(x_n),f(y_n))\to 0$.

Definition 2. [1] The function sequence (f_n) is called *Cauchy alpha convergent* to the function f on X if for every Cauchy sequence (x_n) and (y_n) of points of X with the property $d(x_n,y_n)\to 0$, the sequence $\rho(f_n(x_n),f(y_n))\to 0$.

Definition 3. [2] The sequence $(f_n)\in \mathfrak{sf}(Y^X)$ is called *semi uniform alpha convergent* to $f\in Y^X$ on X and denoted by $f_n \rightrightarrows_{s\alpha} f$ if

1. $f_n\to f$
2. For every $\varepsilon>0$ there exists a $\delta=\delta(\varepsilon)>0$ such that for all $n\in\mathbb{N}$ there exists $m\in\mathbb{N}$ with $m\geq n$ such that for all $x,y\in X$ with $d(x,y)<\delta$ we have $\rho\left(f_m(x),f(y)\right)<\varepsilon$

hold.

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Convergence and Asymptotic Behavior of Mellin-Steklov Exponential Sampling Series in Logarithmically Weighted Spaces

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key-words: Mellin-Steklov integrals, exponential sampling series, weighted approximation.

Abstract:
In this talk, we investigate the approximation properties of r-th order ($r \in \mathbb{N}$) Mellin-Steklov type exponential sampling operators within logarithmically weighted spaces of continuous functions. Initially, we establish the well-definiteness of these operators, demonstrate their pointwise and uniform convergence in weighted spaces, and present the rates of convergence via a suitable modulus of continuity in logarithmic weighted spaces. Subsequently, we establish a quantitative representation of the pointwise asymptotic behavior of these operators using Mellin-Taylor's expansion.

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Approximation of Discontinuous Functions By Kantorovich Exponential Neural Network Operators

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key-words: Kantorovich exponential neural networks operators, jump discontinuity, sigmoidal function.

Abstract:
In this talk, we analyze the behavior of Kantorovich exponential neural network operators when discontinuous signals are considered. First, we establish a fundamental lemma that decomposes the operator by utilizing the critical index at the point of a jump discontinuity. Using this lemma, we prove certain approximation theorems, providing necessary and sufficient conditions for the convergence of these operators at non-removable jump discontinuities. Furthermore, we investigate the conditions under which the approximation is not achieved; specifically, we present a theorem demonstrating that the family of operators fails to converge pointwise at the discontinuity point without strict additional assumptions on the sigmoidal kernel.Finally, some numerical computations are performed to verify the approximation of discontinuous functions f by the proposed operators $K_n^{\chi}{}^{\sigma} f$.

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Fixed Point Results for Rational Type F-Contractions in Perturbed Metric Spaces

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key-words: Fixed point, perturbed mapping, perturbed metric space.

Abstract:
This study investigates fixed point theory within the framework of perturbed metric spaces, a setting that extends classical metric spaces by incorporating systematic perturbations into distance measurements. We introduce a new class of mappings termed "rational type F-contractions," which generalizes Wardowski's F-contractions by utilizing a rational expression in the contraction condition. The main result establishes the existence and uniqueness of fixed points for these mappings in complete perturbed metric spaces, provided the mapping is perturbed continuous. Furthermore, we present illustrative examples and corollaries to demonstrate that our findings unify and extend several existing fixed point theorems in the literature.
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Fixed Point Theorems for Z-Contractions in Perturbed Metric Spaces

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key-words: Fixed point, perturbed mapping, perturbed metric space.

Abstract:
In this presentation, we extend the concept of simulation functions and Z-contractions to the framework of perturbed metric spaces. Perturbed metric spaces, which generalize classical metric structures by incorporating a perturbation term into the standard triangle inequality, provide a more flexible and adaptable setting for fixed point theory. We introduce the notion of Z-contractions in perturbed metric spaces and establish new criteria for the existence and uniqueness of fixed points for such mappings. To demonstrate the validity and effectiveness of our theoretical results, we provide a concrete non-trivial example within a perturbed metric space where standard metric axioms are appropriately relaxed. Ultimately, our findings unify and generalize several existing fixed point theorems in the literature, effectively bridging the gap between simulation functions and perturbed metric structures.

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An Affine ARX-Based Adaptive RST Framework for Disturbed Buck Converter Systems

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key-words: Affine riscrete-time systems, recursive identification, adaptive polynomial control.

Abstract:
This study considers an adaptive polynomial control problem for a nonideal DC–DC buck converter operating under line and load disturbances. Starting from a nonideal averaged continuous-time model, an equivalent affine discrete-time input–output representation is introduced for control design purposes. The proposed model is formulated as a bias-augmented ARX structure, where the additional constant term is used to capture steady-state offsets, operating-point shifts, and lumped nonideal effects that cannot be adequately represented by a low-order linear ARX model alone. This formulation allows a separation between dynamic coefficients and constant offset components, thereby reducing unnecessary parameter drift in online identification. Based on this representation, the converter dynamics are described by time-varying polynomial coefficients that are recursively estimated via a forgetting-factor recursive least-squares algorithm. The estimated coefficients are then used to update, at each sampling instant, the RST controller polynomials through a Diophantine equation associated with a prescribed closed-loop characteristic polynomial. The resulting controller includes an explicit discrete-time integral factor, which ensures offset rejection and asymptotic tracking of constant references under constant disturbances, provided that the closed-loop system remains stable. The proposed framework yields a unified adaptive control scheme in which the bias term improves the identification quality, while the polynomial controller compensates for operating-point variations induced by abrupt input-voltage and load changes. The analysis shows that the affine augmentation is not merely a modeling convenience, but a structural component that improves parameter interpretability and supports smoother online adaptation. The effectiveness of the proposed scheme is evaluated through simulation studies under line and load disturbances.

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Dominants and Their Properties in Lattice Scaled Spaces

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key-words: Dominated Map, ultrametric space, discrete dynamics.

Abstract:
In the present talk, we focus on L-scaled spaces, a class of generalized ultrametric spaces where the distance function $d : M \times M \to L$ takes values in a lattice L possessing a minimal element, see [1]. These spaces provide a robust setting for analyzing the *bo*-convergence of sequences, which lifts the classical *o*-convergence on L to M. The primary focus of this talk is the theory of domination, wherein the behavior of a mapping $f : M \to M$ is governed by an order-theoretic modulus of continuity known as a dominant (or majorant) $S : L \to L$. We establish that for a specific mapping, the collection of all dominants inherits the lattice structure of L and is closed under pointwise order limits. This duality between the mapping's dynamics and the dominants' algebraic properties forms the basis for characterizing σ-*bo*-continuity and the preservation of *bo*-Cauchy sequences. Furthermore, we present several metric inequalities in L that do not hold in classical metric settings.

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Numerical Stability of Branched Continued Fraction Expansions of Lauricella-Saran Hypergeometric Functions F_K and Their Ratios in Special Cases

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key-words: Numerical stability, backward recurrence algorithm, branched continued fraction, Lauricella-Saran's hypergeometric function F_K, approximation by rational functions.

Abstract:
The numerical stability of the backward recurrence algorithm for computing approximants expansions of the Lauricella-Saran hypergeometric functions F_K and their ratios in special cases into branched continued fractions is considered. New estimates of the relative errors of approximants computations are obtained, showing the dependence of the error of the approximant on the magnitude of the rounding errors of its elements, and the values of the coefficients of the branched continued fractions. New sufficient conditions for the sets of numerical stability for the aforementioned approximants. are also established. Numerical experiments allowed us to evaluate the practical effectiveness of the proposed theoretical results.

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Some Properties of Generalized Degenerate Tangent Polynomials and Their Associated Matrices

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key-words: Generalized degenerate tangent polynomials, tangent polynomials, ∇_λ-Appell sequences.

Abstract:
We introduce and study generalized degenerate tangent polynomials $\mathbb{T}_{s,\lambda}^{(\alpha)}(\xi)$ and their associated matrices using the generating function approach. Using this framework, we derive their main algebraic and differential properties, including their characterization as Δ_λ-Appell sequences, recurrence relations, inversion formulas, and addition formulas. We also obtain new factorizations and matrix identities connecting generalized degenerate Pascal matrices, tangent polynomial matrices, and Fibonacci matrices. Numerical experiments further illustrate the symmetry and distribution of their zeros in the complex plane.

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Structural Stability and Rational q-Deformations in Parikh q-Determinants

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key-words: Parikh q-matrix, q-Stirling numbers, Hessenberg matrix, Euler operator.

Abstract:
In this study, we investigate the degree mismatch phenomenon arising in the classical q-Stirling expansions of Parikh q-determinants associated with periodic words. For determinant families generated by periodic structures of the form $w = (ba)^n$, the synchronization of the determinant sequence with q is introduced and its analytic extension modeled via the Euler differential operator $T = q\frac{d}{dq}$ is examined. Then, the connections between these analytic structures and weighted Hessenberg-type matrices are addressed and various structural stability constraints are derived in this context. In particular, the degree-suppressing effects of lexicographical inversions ($b \to a$) on the determinant behavior are investigated. Using a structural asymmetry index (τ), we propose structural stability criteria describing when periodic structures remain inside the polynomial ring $\mathbb{Z}[q]$ and when rational q-deformations arise in the field $\mathbb{Q}(q)$. These results establish new connections between Parikh q-determinants, operator methods, generating functions and structural stability phenomena in combinatorics on words.

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On the Zero Behavior of Degenerate Bernstein Basis Functions

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key-words: Degenerate Bernstein basis functions, Bernstein basis functions, zeros and critical points.

Abstract:
The distribution of zeros of a polynomial plays a central role in characterizing its structural and analytic properties, particularly in relation to how these features evolve under parameter variations. This study focuses on deformations of Bernstein polynomials induced by a deformation parameter λ, interpolating between classical and generalized settings. Within this framework, we analyze the zero distribution of degenerate Bernstein basis functions in the sense of T. Kim and D. Kim [4, 5], together with the behavior of their associated critical points. We establish sharp localization results, showing that these zeros lie in explicitly described λ-dependent intervals. In addition, we prove a precise interlacing between zeros and critical points and characterize its evolution as λ varies. These results provide a quantitative description of the behavior of zeros and critical points with respect to the deformation parameter.

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Approximation Properties of Modified Neural Network Operators Activated by Sigmoidal Functions

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key-words: Neural network operators, sigmoidal functions, modulus of continuity.

Abstract:
In this study, we introduce and investigate a novel family of linear neural network operators activated by sigmoidal functions and modified by a suitable function ρ. This modification yields a non-uniform sampling scheme, enabling the approximation process to adapt effectively to the local behavior of the target function. We establish the fundamental approximation properties of these operators, proving both pointwise and uniform convergence on compact intervals. Furthermore, a quantitative error estimate is derived via the modulus of continuity, revealing that the rate of convergence is governed by the smoothness of the transformed function $f \circ \rho^{-1}$. We also establish a Voronovskaya-type asymptotic formula that explicitly characterizes the local approximation error in terms of the first and second derivatives of ρ. Finally, numerical examples are presented to validate the theoretical findings and demonstrate the efficacy of the proposed operators in selective approximation tasks.

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Convergence of Multidimensional Mellin Type Singular Integral Operators with Non-Isotropic Kernels

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key-words: Linear singular integral, Non-isotropic distance, multidimensional Mellin convolution integral, convergence, rate of convergence.

Abstract:
Here we give some approximation theorems concerning pointwise convergence and rate of pointwise convergence of multidimensional Mellin type singular integral operators of the form

$$(T_w f)(\mathbf{s}) = \int_{\mathbb{R}^n_+} K_w(\mathbf{t})\, f(\mathbf{st}) \frac{d\mathbf{t}}{\langle \mathbf{t} \rangle}, \qquad w > 0,$$

where the kernel function K_w satisfies some singularity assumptions with non-isotropic kernels.

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Mid p-Summable Sequences and Applications to Lipschitz Operators

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key-words: Banach space sequences, Lipschitz operators, summing operators.

Abstract:
Mid p-summable sequences were introduced by Karn and Sinha [3]. This notion naturally emerges as an extension of bounded sets within the framework of p-summability. As a new type of summability for vector-valued sequences, mid p-summability has attracted significant attention in the literature and has been studied extensively; see, for example, [2, 1, 5, 4].

Let E be a Banach space and let $1 \leq p < \infty$. A sequence (x_n) in E is said to be *norm p-summable* if $\sum_{n=1}^{\infty} \|x_n\|^p < \infty$, and it is called *weakly p-summable* if $\sum_{n=1}^{\infty} |x^*(x_n)|^p < \infty$ for all $x^* \in E^*$. Every norm p-summable sequence is weakly p-summable; however, the converse does not hold in general and is valid only when E is finite-dimensional. The spaces $\ell_p(E)$ and $\ell_p^w(E)$, consisting of norm and weakly p-summable sequences, respectively, are Banach spaces when endowed with their natural norms. A sequence $(x_j)_{j=1}^{\infty}$ in a Banach space E is called *mid-p-summable* if, for every sequence $(x_n^*)_{n=1}^{\infty} \in \ell_p^w(E^*)$, one has $\left((x_n^*(x_j))_{j=1}^{\infty}\right)_{n=1}^{\infty} \in \ell_p(\ell_p)$. The collection of all such sequences is denoted by $\ell_p^{\mathrm{mid}}(E)$. This notion provides an intermediate structure between classical summability concepts, as reflected in the inclusions $\ell_p(E) \subseteq \ell_p^{\mathrm{mid}}(E) \subseteq \ell_p^w(E)$.

In this work, a new class of Lipschitz operators is introduced by means of mid p-summable sequences, and several fundamental properties of this class are investigated. Moreover, further results obtained via mid p-summable sequences in the setting of Lipschitz operators are presented.

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Numerical Spectral Analysis of a Chafee–Infante Type Equation on Unbounded Domains

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key-words: Spectral method, Chafee–Infante equation, rational approximation, unbounded domains, weighted Sobolev spaces, Arar–Boulmezaoud functions.

Abstract:
This paper presents a rational spectral method for the numerical approximation of a nonlinear Chafee–Infante type equation posed on the whole real line. The proposed approach relies on the Arar–Boulmezaoud basis functions, which are specifically adapted to unbounded domains and naturally incorporate decay properties at infinity. A weak variational formulation is established in a weighted Sobolev framework, and the resulting nonlinear Galerkin system is discretized using spectral techniques combined with Gauss-type quadrature formulas. The semi-discrete system is integrated in time through a suitable numerical scheme, and the nonlinear term is treated using an iterative procedure. Numerical experiments are performed to validate the accuracy, stability, and spectral convergence of the proposed method on large computational domains.

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On Exponential-Type Sampling Series: Upper Bounds of Operator Differences

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key-words: Exponential-type sampling series, upper bounds for operator differences, Kantorovich-type operators.

Abstract:
In this talk, we present results that provide upper bounds for operator differences in logaritmic weighted spaces of functions. To this end, we consider the following operators: generalized exponential sampling operators [2] and their Kantorovich [3] and generalized Kantorovich forms [1]. Finally, we also discuss several kernel examples that support our theoretical results. It is worth emphasizing that the present study adopts a direct approach within the framework of Mellin analysis for the processes under consideration.

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A Mixture Model Using Unit Type Lindley Distributions

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key-words: EM algorithm, unit-Lindley distribution, mixture model.

Abstract:
This paper proposes a parsimonious three-parameter mixture model for data supported on the unit interval. The model combines two structurally different unit-Lindley transformations arising from the same Lindley parent distribution. Its novelty lies not in the use of a finite mixture itself, but in bringing these two complementary Lindley-based transformations together within a simple bounded model. This construction preserves the simplicity of one-parameter components while allowing symmetric, asymmetric, sharply peaked, and bimodal shapes. Closed-form expressions are obtained for the density and distribution functions, and several distributional properties are derived, including moments and stress–strength reliability. For the basic three-parameter model, parameter estimation is carried out by the Expectation–Maximization algorithm, and large-sample inference is based on the observed-data information matrix obtained through the missing-information principle. The main empirical application to piped-water access proportions shows that the proposed model can improve likelihood-based fit while yielding interpretable latent profiles. A limited covariate-dependent example is also included only as an illustrative extension.

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Convergence Domains of Branched Continued Fraction Expansions of Ratio $H_4(a, d+1; c, d; \mathbf{z})/H_4(a+1, d+1; c, d+1; \mathbf{z})$

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key-words: Branched continued fraction, Horn hypergeometric function, convergence.

Abstract:
The Horn's hypergeometric function H_4 is defined by (see [2])

$$H_4(a, b; c, d; \mathbf{z}) = \sum_{k,l=0}^{\infty} \frac{(a)_{2k+l}(b)_l}{(c)_k(d)_l} \frac{z_1^k}{k!} \frac{z_2^l}{l!}, \quad |z_1| < p, \ |z_2| < q,$$

where a, b, c, and d are complex numbers herewith $c, d \notin \{0, -1, -2, \ldots\}$, $(\cdot)_n$ is the Pochhammer symbol, p and q are positive numbers such that $4p = (q-1)^2$ herewith $q \neq 1$, $\mathbf{z} = (z_1, z_2) \in \mathrm{C}^2$.

From Theorem 1 in [1], under the conditions that $b = d + 1$ and $(ij)_0 = (1, 2)$, we get that the function

$$\frac{H_4(a, d+1; c, d; \mathbf{z})}{H_4(a+1, d+1; c, d+1; \mathbf{z})} \tag{2}$$

has a formal branched continued fraction expansion of the form

$$1 - \frac{d-a}{d} z_2 - \frac{h_1 z_1}{1 - z_2} - \frac{h_2 z_1}{1 - z_2} - \frac{h_3 z_1}{1 - z_2} - \cdots, \tag{3}$$

where

$$h_1 = \frac{2(a+1)}{c}, \ h_k = \frac{(2c - a + k - 3)(a + k)}{(c + k - 2)(c + k - 1)}, \ k \geq 2.$$

We consider the problem of establishing the convergence criteria of the branched continued fraction expansion 3 of the ratio 2. To solve it, the technique of expanding the domain of convergence of the branched continued fraction from the known small domain of convergence to a wider domain of convergence is used. Under various conditions on the parameters of the Horn hypergeometric function H_4, we study different domains of convergence in various unbounded domains of the space $\mathbf{C}^2$.

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Pseudospectra in Banach Algebras and Banach Jordan Algebras

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key-words: Banach Jordan algebra, quadratic pseudospectrum, numerical range.

Abstract:
This paper provides a study of pseudospectra in Banach algebras and Banach Jordan algebras, systematically transitioning from the classical associative setting to the intrinsically Jordan quadratic framework. Classical pseudospectra are defined via the resolvent, while the Jordan analogue, the quadratic pseudospectrum $\sigma_\epsilon^{(2)}(a)$, is formulated using the quadratic operator U_x which intrinsically governs invertibility in Jordan algebras. Leveraging Moreno's localization theorem which embeds singly generated Jordan subalgebras into full associative commutative Banach subalgebras we establish a universal inclusion $\sigma_\epsilon^{(2)}(a) \subseteq \sigma_\epsilon(a)$ valid for all Banach Jordan algebras. For self-adjoint elements in JB- and JB^* -algebras, we recover exact equivalence,

$$\sigma_\epsilon^{(2)}(a) = \{\lambda : dist(\lambda, \sigma_A(a)) \leq \epsilon\}$$

and derive direct and inverse quadratic pseudospectral mapping theorems under local injectivity. Moreover, we demonstrate that the numerical range controls quadratic resolvent growth, yielding Kreiss-type estimates and pseudospectral radius bounds in the JB^* -setting. A critical assessment clarifies that exact equality is a C^* -type phenomenon rather than a universal feature, and the numerical range control for general non-self-adjoint elements remains an open problem. Overall, this synthesis validates the quadratic pseudospectrum as the natural stability object in Jordan theory, structurally compatible with classical pseudospectral analysis in all C^* -related contexts.

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Korovkin and Voronovskaya Results for Positive Bivariate Operators under Generalized Convergence

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key-words: Korovkin, Voronovskaya, bivariate operator.

Abstract:
This talk is based on a natural continuation of papers [1] and [2]. In [1], the author deals with a qualitative Korovkin-type approximation result about the convergence of sequences of functions defined by means of linear operators, where the limit of the approximation process is the image of a function under certain differential operator, and where a general notion of convergence is utilized. Then, the study is completed by analyzing conditions that guarantee the existence of an asymptotic formula. On the other hand, in [2], the authors prove the corresponding quantitative result, and study the saturation class with the help of that asymptotic formula. While both previous papers were written under a univariate setting, the present one goes into the bivariate case.

The main achievement of this work comes from the fact of considering a general notion of convergence, as it was also the case with [1, 2]. There is a long list of papers where different notions of convergence of double sequences of functions are moved to the setting of approximation theory. Particular double sequences of linear operators are analyzed under particular concepts of convergence, and then, the usual topics within that theory are studied. The framework that we set in this paper, and the results we prove, not only cover all those particular situations, and many previous results in the subject, but provide as well a direct way to deal with further ones that may arise in the future.

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Bridging Extending and Supplemented Modules: Towards a Unified Framework in Module Theory

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key-words: Extending modules, supplemented modules, bridging modules.

Abstract:
This paper presents a unified theoretical framework connecting extending modules, supplemented modules, and bridging modules. Its main objective is to rigorously investigate their structural relationships, identify underlying similarities, and establish homological conditions under which these classes of modules can be considered equivalent in a categorical sense.
The study adopts a pure mathematical approach based on formal reasoning through lemmas and theorems, supported by explicit constructions and counterexamples. It further highlights new algebraic relations in connection with classical results while addressing existing conceptual gaps.
The proposed framework is illustrated through examples involving $\mathbb{N}$-modules, Artinian rings, and semisimple rings, demonstrating both its applicability and limitations. A key result shows that, under suitable structural and homological conditions, extending and supplemented modules arise as special cases of bridging modules.
This unification provides deeper insight into module decomposition, exact sequences, and classification theory within homological algebra, offering potential extensions for categorical and computational approaches.

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A θ-Scheme for Complex Nonlinear Klein-Gordon Equations in the Nonrelativistic Limit: Stability Analysis and Numerical Results

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key-words: θ-scheme, complex nonlinear Klein-Gordon equation, stability analysis.

Abstract:
The nonlinear Klein-Gordon equation (NKGE) stands as a fundamental model in relativistic quantum mechanics for describing spin-$\frac{1}{2}$ particles, and it can be regarded as a relativistic version of the Schrödinger equation [2]. Beyond its theoretical origins, the NKGE has emerged as a pivotal tool in nonlinear optics and plasma physics, where it governs the evolution of robust wave structures and complex multi-scale phenomena in the nonrelativistic limit regime [1]. In this study, we investigate the NKGE within the nonrelativistic limit regime, formulated as

$$\varepsilon^2 \partial_t^2 u = \Delta u - \frac{1}{\varepsilon^2} u - \lambda |u|^2 u, \qquad x \in \Omega \subseteq \mathbb{R}, \quad 0 < t \leq T, \tag{1}$$

where u is a complex-valued unknown function and $0 < \varepsilon \ll 1$ denotes a small scaling parameter. Here, $\lambda \neq 0$ is a real constant independent of ε, and T represents a fixed final time [2]. Furthermore, some suitable initial and boundary conditions must be specified for Eq. (1). This regime is characterized by rapid oscillations in both time and space, which present significant challenges for standard numerical approximations. While existing literature often struggles with the computational cost of nonlinear iterations or the rigorous analysis of complex-valued solutions, our work addresses these issues by decoupling the real and imaginary components and employing an efficient linearization technique. We propose a versatile θ-scheme for $\theta \in [0, 1]$, which naturally incorporates the explicit method ($\theta = 0$), the Crank-Nicolson (CN) method ($\theta = 1/2$), and the implicit method ($\theta = 1$) as special cases. A core contribution of this research is a deep theoretical investigation used to derive conditional stability bounds for the entire range of θ, ensuring the reliability of the method across different parameter regimes. A mild condition mentioned in our theorem does not limit the applicability of the method. Numerical experiments, conducted using the discrete L_2 norm, are provided to evaluate the performance of the proposed schemes. As usual, the explicit and implicit schemes provide first-order temporal accuracy. However, the CN scheme exhibits first-order accuracy in time, contrary to the usual second-order behavior. To recover second-order temporal convergence, we propose a three-level θ-scheme.

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4th International Conference: Constructive Mathematical Analysis

Convergence and Smoothness of the Mehler-Fock Transform and Application to Overdetermined Boundary Value Problems in $\mathbb{H}^n \times \mathbb{R}$

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key-words: Overdetermined problem, integral transform, elliptic pde.

Abstract:
Let $\Omega \subset \mathbb{R}^n$ denote an unbounded smooth domain. Berestycki, Caffarelli, Nirenberg in [1] studied the following boundary value problem

$$\begin{cases} \Delta u + f(u) = 0 & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega \\ \frac{\partial u}{\partial \nu} = const & \text{on } \partial\Omega, \end{cases} \tag{1}$$

where f is a Lipschitz function and ν is the outward unit normal vector field.

They considered domains bounded by the graph of a C^2 Lipschitz function F:

$$\Omega := \{x \in \mathbb{R}^n \mid x_n \geq F(x_1, x_2, \ldots, x_{n-1})\}.$$

More precisely, if for any $p \in \mathbb{R}^{n-1}$, uniformly, $\lim_{x \in \mathbb{R}^{n-1}, |x| \to \infty} (F(x+p) - F(x)) = 0$ and f satisfies some hypotheses, then if there exists a bounded positive solution of (1), the authors showed that Ω is a half-space, that is F is a constant function.

In the same work the authors stated the following conjecture: if Ω is a smooth domain such that $\mathbb{R}^n \setminus \Omega$ is connected and there exists a bounded positive solution of (1) for some Lipschitz function f, then Ω is a half-space, a ball, the complement of a ball, or a circular-cylinder-type domain: $\mathbb{R}^j \times B$ being B a ball. Even though the conjecture is known to be false in the general case, the case when the domain is an epigraph has attracted special attention because of its relation with De Giorgi and Bernstein conjectures.

Overdetermined elliptic problems have been studied in the case when the ambient manifold is a Riemannian manifold as well: the unit sphere, the hyperbolic space, $\mathbb{H}^N \times \mathbb{R}$ and $\mathbb{S}^N \times \mathbb{R}$.

In [2] we showed that under the hypotheses $F \in C^{2,\alpha}$, $f \in C^{1,\alpha}$, $f'(0) \leq 1/4$, there exist unbounded domains Ω (bounded by the vertical graph of a nonconstant function) in $\mathbb{H}^2 \times \mathbb{R}$ for which (1) admits a positive solution. Such domains and the corresponding solutions are obtained by perturbing the half-space $\mathbb{H}^2 \times \mathbb{R}^+_0$ and the solution to $u''(z) + f(u(z)) = 0$ such that $u(0) = 0$, $u(z) \to \pm 1$ as $z \to \pm\infty$. The proof is based on the use of the Mehler-Fock transform. In other terms the Berestycki-Caffarelli-Nirenberg conjecture is false in the case when the functions f, F have derivatives which are Holder functions.

In our recent work we prove the same result in $\mathbb{H}^n \times \mathbb{R}$.

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4th International Conference: Constructive Mathematical Analysis

A Constructive Definition of the Improper Riemann Integral on a Separable Banach Space

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key-words: Improper Riemann integral, Lebesgue integral, separable Banach space.

Abstract:
The goal of this talk is to construct an improper Riemann integral on a separable Banach space which possesses all of the fundamental properties of the improper Riemann integral on $\mathbb{R}^n$. Let $\mathcal{B}$ represent a separable Banach space. Using an idea from Yamasaki [3], the paper [1] presents a proof that $\mathcal{B}$ has an isomorphic, isometric embedding in $\mathbb{R}^\infty = \mathbb{R} \times \mathbb{R} \times \cdots$. In this work we will use this embedding to define an improper Riemann integral on $\mathcal{B}$ that extends the definition of the Riemann integral on a special rectangle in $\mathcal{B}$ as discussed in [2]; which makes the derivations of most of its properties virtually identical to those of its finite-dimensional analogue.

Similar to the Lebesgue integral on $\mathcal{B}$, this improper Riemann integral is constructive in the sense that it equals a limit of improper Riemann integrals on $\mathbb{R}^n$ as $n \to \infty$. We will use this convergence to extend some well-known probability density functions to $\mathcal{B}$ and define a Fourier transform on $\mathcal{B}$. This talk will conclude by discussing some ways in which this improper Riemann integral advances the theory of Lebesgue integration on $\mathcal{B}$.

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On the Representation of Lauricella-Saran Hypergeometric Functions F_M and Their Ratios in Special Cases by Branched Continued Fractions

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key-words: Hypergeometric function, branched continued fraction, analytic function, approximation by rational functions, convergence, analytic continuation.

Abstract:
The problem of approximating Lauricella-Saran hypergeometric functions F_M and their ratios in special cases by a special family of functions – branched continued fractions – is considered. Under certain conditions of the parameters of the Lauricella-Saran hypergeometric functions F_M, the domains of analytic continuation of the above-mentioned functions are established using their expansions into branched continued fractions, the elements of which are polynomials of three complex variables. Numerical experiments allowed us to evaluate the practical efficient approximating analytic functions by branched continued fractions.

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A Randomized Shifted Finite Volume Method to Break Resonance Locking in 1D Homogenization

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key-words: Homogenization, finite volume method, resonance locking, random shift, discontinuous coefficients, continuous coefficients.

Abstract:
We investigate the numerical approximation of a one-dimensional elliptic homogenization problem with highly oscillatory periodic coefficients. When the mesh size h is an integer multiple of the period ε ($h/\varepsilon \in \mathbb{N}$), the classical finite volume method suffers from resonance locking: the error does not decrease under mesh refinement. We propose a simple non-intrusive fix: randomly shift the evaluation points of the coefficient at each interface by a small amount uniformly distributed in $[0, h]$. This infinitesimal perturbation breaks the artificial periodicity of the discrete operator. Numerical experiments are performed for both discontinuous (two-phase media with contrasts up to 100) and continuous ($a(y) = 1 + 0.5\sin 2\pi y$) coefficients, always keeping $h = \varepsilon$. For discontinuous coefficients, the classical pointwise scheme locks (error $\sim 5.5 \times 10^{-2}$), while harmonic averaging (requiring knowledge of discontinuities) achieves second-order accuracy (7.8×10^{-5} at $N = 2560$). Our random shift restores first-order convergence and reduces the error by factors of 30–60 compared to the locked scheme, making it a robust alternative when microstructural details are unknown. For continuous coefficients, harmonic averaging gives no advantage; the random pointwise method again yields first-order convergence and reaches an error of 7.6×10^{-4} at $N = 4096$, which is a gain of 25.5 over the locked classical scheme. The method is trivial to implement, requires no microstructure knowledge, and extends directly to higher dimensions. This randomized shift thus provides a cheap and effective remedy against resonance locking in multiscale computations. The model problem is

$$-\frac{d}{dx}\Big(a\big(\frac{x}{\varepsilon}\big)\frac{du}{dx}\Big) = 1, \quad x \in (0,1), \quad u(0) = u(1) = 0, \tag{1}$$

with a periodic. For $h = \varepsilon$ the classical scheme reduces to a constant-coefficient discrete operator, explaining the locking. The random shift introduces variation and restores convergence.

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Fixed Point Theorems in Digital Metric Spaces and Applications in Fractal Image Compression

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key-words: Digital metric space, fractal image compression, vector-valued metric.

Abstract:
Fixed point theory has long been recognized as a powerful tool across various fields, from functional analysis to computer science [1, 3]. However, when problems are considered in a digital setting, such as image analysis, the continuous framework must be adapted to discrete structures and adjacency relations [1]. Digital metric spaces provide a natural environment for extending fixed point results to model such discrete data [1]. This seminar will comprehensively explore next-generation contraction mappings developed within digital metric spaces, specifically focusing on digital F-contractions [1], rational type common fixed point theorems [3, 4], and θ-contractions [2]. Moving beyond traditional scalar approaches, the concept of vector-valued digital metric spaces will be introduced to address multi-objective optimization problems, such as simultaneously measuring pixel intensity and structural gradients [4, 2]. In addition to the theoretical foundations, the practical efficacy of these mathematical frameworks will be demonstrated through their applications in fractal image compression [3, 4, 2]. Specifically, the seminar will showcase how fractal-based models around the digital Sierpinski triangle [3] and the novel Adaptive Digital F-Fractal Compression (ADFFC) algorithm [4] successfully optimize the trade-off between image fidelity and compression ratio using fixed point iterations.
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Approximation by a Novel Modification of a Mellin-Type Kernel

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key-words: Logarithmic moments, Mellin derivative, Euler-Gamma function.

Abstract:
In this presentation, we express the approximation features for a novel modification of the Mellin-type kernel. Moreover, a Voronovskaya-type theorem is given using the Euler Gamma function and Mellin derivatives.

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Hyperbolic Wavelets, Basis Problem

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key-words: Hyperbolic wavelets, basis problem in the disc algebra.

Abstract:
We study the explicit hyperbolic Malmquist–Takenaka system associated with the logarithmic hyperbolic grid in the unit disc. We proved that the corresponding multiresolution levels form a complete model set in the disc algebra $A(\mathbb{D})$; equivalently, the union of the finite resolution spaces is dense in $A(\mathbb{D})$. We then investigate the convergence of the orthogonal projections in the supremum norm. A direct kernel-based approach leads to a multiscale shell decomposition. This yields uniform control of the low-frequency and bounded-width critical-frequency contributions, but also shows that absolute L^1-kernel estimates are too crude to capture the cancellation present in the high-frequency regime. Motivated by this obstruction, we develop two alternative reductions of the max-norm convergence problem. The first is operator-theoretic: using the interpolation property of the projections and the exact Christoffel–Darboux formula for the detail kernels, we reduce the problem to an off-scale decay estimate for the detail operators $D_j = P_j - P_{j-1}$. The second is sampling-based: using the geometry of the hyperbolic lattice, we reduce convergence to a uniform norming-set inequality for the finite resolution spaces. As a rigorous partial result, we prove max-norm convergence for all functions whose detail contributions are summable in the supremum norm.

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A Smooth Lagrangian-Based Neural Network for Solving Semidefinite Programming

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key-words: Semidefinite programming, neural network, augmented lagrangian smoothing.

Abstract:
In this talk, we introduce a smooth gradient-based neural network for semidefinite programming (SDP) using the augmented Lagrangian smoothing (ALS) mapping. The proposed model is formulated through a continuous energy function whose equilibrium points correspond to the Karush–Kuhn–Tucker (KKT) conditions of the SDP problem. We present the main theoretical results, including well-posedness, Lyapunov stability, global convergence, and local exponential convergence. Numerical experiments on benchmark problems show that the method converges reliably and performs competitively compared with existing smoothing-based neural network approaches. In particular, while the proposed method has comparable accuracy on eigenvalue optimization problems, it gives substantially better results for spectral norm minimization. The framework also introduces a penalty parameter ρ that provides additional flexibility in balancing primal and dual variables.

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Rational Approximation and Meromorphic Continuation

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key-words: Rational approximation, meromorphic continuation, Walsh table.

Abstract:
New asymptotic results for products of best rational approximation errors taken from several rows of the Walsh table will be presented. These results connect the rate of rational approximation with radii of meromorphic continuation, extending classical theorems of Hadamard and Saff–Gonchar. The proof uses Hankel operators and an AAK-type theorem.

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Degenerate Appell Sequences via Caputo-Type Fractional Difference Operator

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key-words: Caputo operator, Mittag-Leffler function, ∇_λ-Appell sequences.

Abstract:
We introduce the notion of ∇_λ^α-Appell sequences of functions based on the discrete fractional Caputo operator and the degenerate Mittag–Leffler function. We analyze several properties of these function sequences, including explicit expressions for each term and their associated generating functions. In particular, we establish some basic novel properties for the case $\alpha = 1$, in which the corresponding sequences become ∇_λ-Appell polynomial sequences.

It is worth noting that the use of fractional operators and expansions in fractional powers which mimic the structural form of Appell (or Scheffer) sequences is a recent development (cf. [1, 2] and the references therein). To the best of our knowledge, the novelty of this work has been to study an analogous situation incorporating the notion of the degeneracy (cf. [3, 4, 5]).

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Inequalities for Complex Polynomials

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key-words: Polynomials, zeros, inequalities.

Abstract:
A result concerning the inequalities for complex polynomials, credited to Rivlin [1], states that for a polynomial $P(z)$ of degree n that has no zeros in the disc $|z| < 1$, the following inequality holds:

$$\max_{|z|=r} |P(z)| \geq \left(\frac{1+r}{2}\right)^n \max_{|z|=1} |P(z)|,$$

for $0 \leq r \leq 1$. Rivlin's inequality has been generalized to encompass discs of radius greater than or equal to 1 by several mathematicians. In this presentation, we discuss more refined and generalized inequalities, of which a specific case would enhance the classical Rivlin's inequality.

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Spherical Number Algebras: An Invariant-First Framework from Conceptual Architecture to Symbolic and Numeric Presentations

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key-words: Spherical number algebras, point-free mathematics, invariant structures.

Abstract:
This paper introduces Spherical Number Algebras (SNA), a new structural framework in pure mathematics founded on the principles of admissibility-first, invariant-first, and point-free reasoning. Rather than assuming primitive objects, fields, numbers, or spatiotemporal substrates, SNA takes admissible assembly-events as its fundamental entities and treats structural admissibility as logically prior to composition. Each admissible event carries an invariant package from which observables, symbolic descriptions, and numerical representations arise as derived projections.

The framework is organized around four guiding principles—admissibility before composition, invariants before observation, articulation before objecthood, and point-free continuity before numeric representation—providing a pre-numeric mathematical foundation in which algebraic and symbolic structures emerge through successive translation layers.

The development proceeds in three parts. Part I establishes the conceptual foundations of admissible assembly-events, invariant packages, and observation as a derived process. Part II develops a point-free pre-numeric formal core based on word rewriting, proving that observable assemblies form canonical quotient structures with well-defined composition, induced invariant projections, and associativity up to observation. Part III introduces symbolic, discrete, and numeric translation layers through coding maps from stabilized admissible events or their observable quotients, where rewriting systems operate on symbolic codes and numerical values arise only as tertiary projections.

Throughout the framework, invariant packages remain primary, while symbolic and numerical descriptions are treated as representational layers rather than ontological primitives. SNA therefore provides an invariant-centered mathematical foundation preceding conventional numeric, metric, probabilistic, or energetic formulations.

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Sums as Sets

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key-words: Set-theoretic calculus, foundations of calculus, dimensional analysis.

Abstract:
This paper develops a set-theoretic account of calculus in which sums are treated as sets before they are treated as numbers. The motivating contrast is between numerical difference and set difference: while an expression such as $f(x) - f(x-h)$ becomes problematic when the width collapses, the corresponding set-theoretic difference remains meaningful, as in $(a,b] \setminus (a,b) = \{b\}$. This paper develops calculus by preserving this exact set-theoretic difference instead of first projecting to numbers.

Setup.
Let f be a set of tuples over R (for concreteness, $R = \mathbb{R}$ suffices), with last coordinate designated as abscissa. Write $f_{\leq b}$, $f_{<b}$, and $f_{=b}$ for the parts whose designated coordinate is respectively at most, less than, or equal to b. The basic decomposition is $f_{\leq b} = f_{<b} \sqcup f_{=b}$, hence $f_{\leq b} \setminus f_{<b} = f_{=b}$.

Evaluation.
Define evaluation as abscission of f at b as $|^b f := f_{\leq b}$. Further, $|^b_a f := |^b f \setminus |^a f$.

Example.
Our running example is simplices: $t_n := \{x \in [0,1]^n \mid x_1 \leq ... \leq x_n\}$, and $t := t_1$.

Differential.
We define differential as set difference $\Delta^b f := f_{\leq b} \setminus f_{<b}$. Thus $\Delta^b f = f_{=b}$. In one dimension, a differential is a singleton; in two dimensions, it is a strip, i.e. a one-dimensional part crossed with a singleton. For example, $\Delta^b t = \{b\}$, $\Delta^b t_2 = [0,b] \times \{b\}$, and $\Delta^1 t_n = t_{n-1} \times \{1\}$.
textbfSums.
We define the sum Σf as $(\Sigma f)_{=x} := |^x f \times \Delta^x t$. Equivalently, $\Sigma f = \bigsqcup_x (|^x f \times \{x\})$. We also define $\Sigma^b_a f := |^b_a (\Sigma f)$, and $\Sigma^b f := |^b (\Sigma f)$. For example, $\Sigma^1 t_n = t_{n+1}$. So, linear f makes Σf a triangle ($\frac{square}{2}$), as expected. Similarly, $\Sigma\Sigma f$ is a triangular pyramid ($\frac{cube}{6}$), as expected.

Quotient.
Cartesian quotient is inverse to Cartesian product: $C = A \times B \implies C/B = A$.

Derivative.
We define the derivative to be the Cartesian quotient of an exact differential. $D^b f := \Delta^b f / \Delta^b t$. Equivalently, $D^b f = \Delta^b f / \{b\}$. Further, define $DF := f \iff D^b F = |^b f$.
For example, $D^1 t_n = t_{n-1}$.

Fundamental Theorems of Calculus (FTC).
Part 1: $D\Sigma f = f$. Any anti-sum (f) is the derivative: $F = \Sigma f \implies f = DF$.
Part 2: $\Sigma DF = F$. Any anti-derivative (F) is the sum: $f = DF \implies F = \Sigma f$.
Combining 1 & 2, is the unified fundamental theorem: $f = DF \iff F = \Sigma f$.

Example.
For a finite example, let $R = \{1,2,3\}$. Let $f = \{(1),(2),(3)\} \subseteq R^1$.
The evaluations are $|^1 f = \{(1)\}$, $|^2 f = \{(1),(2)\}$, and $|^3 f = \{(1),(2),(3)\}$.
The differentials are $\Delta^1 f = \{(1)\}$, $\Delta^2 f = \{(2)\}$, and $\Delta^3 f = \{(3)\}$.
The sum is $\Sigma f = \bigsqcup_x \big(|^x f \times \{x\}\big) = \{(1,1),(1,2),(2,2),(1,3),(2,3),(3,3)\}$.
FTC1 states: $D\Sigma f = f$. Here, $\Delta^2(\Sigma f) = \{(1,2),(2,2)\}$. $D^2(\Sigma f) = \{(1),(2)\} = |^2 f$.

FTC2: start with $R = \{1, 2, 3\}$ and $f = \{(1), (2), (3)\}$. For $x \in R$ assume $D^x F = |^x f$. This forces $F = \{(1, 1), (1, 2), (2, 2), (1, 3), (2, 3), (3, 3)\}$, which equals Σf (see above). So, $\Sigma f = F$.

Summary.
This paper replaces infinitesimal or limiting differentials with exact set-theoretic differentials. A differential is a literal subset; differentiation is Cartesian quotient; and integration is union. Numerical measures may be introduced later, but they are optional representations of a structure already present at the level of sets.

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On the Oscillatory Behavior of Cesàro Means for Fourier-Jacobi Series

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key-words: Fourier-Jacobi series, Cesàro means, dyadic increments.

Abstract:
We investigate oscillatory properties of Cesàro means $\{\sigma_n^\delta(f,x)\}_{n\geq 0}, (\delta > -1)$, for Fourier-Jacobi expansions and their connection with r-variation norms, motivated by recent developments in variation estimates for Fourier series and Carleson-type operators [3]. Using Abel-type transformations and kernel estimates, we derive increment formulas and introduce a near-far decomposition for dyadic increments. We prove that the far contribution is controlled by the maximal partial sum operator, yielding L_p-bounds for dyadic consecutive increment functionals, in the classical mean convergence range for Fourier-Jacobi expansions [2]. We further analyze the kernel structure of the associated dyadic increment operators and identify the near-diagonal region as the principal source of difficulty in obtaining full variation estimates. The results provide a structural framework for future study of variation-norm estimates for Cesàro means of Fourier-Jacobi series.

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Modeling Bounded Data for the Unit Transformed Inverse Weibull Distribution: Estimation and Application

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key-words: Bonferroni and Lorenz curves, bounded distribution, quantile function, maximum likelihood estimation, Monte Carlo simulation.

Abstract:
In this study, we propose a new bounded distribution, called as unit transformed inverse Weibull distribution. The proposed model is defined on the unit interval $(0, 1)$ and is derived from the transformed inverse Weibull distribution defined on $(0, \infty)$. The proposed model extends the flexibility of classical bounded distributions and is particularly suitable for modeling proportion data. Several fundamental statistical properties of the unit transformed inverse Weibull distribution are derived, including the probability density, cumulative function and hazard rate distribution functions, quantile function, order statistics, and entropy, etc. The proposed distribution is characterized are further examined through, the Bonferroni and Lorenz curves. Several estimation methods are employed to estimate the unknown model parameters. The performance of these estimators is evaluated through an extensive Monte Carlo simulation study based on bias, mean squared error, mean absolute bias, and mean relative error criteria. An application to a real data set is presented to illustrate the practical relevance and modeling capability of the proposed distribution.

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Reliability Evaluation for the Linear or Circular Consecutive k-out-of-n: G System Using Continuous Time Bayesian Network

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key-words: Reliability analysis, continuous time Bayesian Network, consecutive k-out-of-n: G system.

Abstract:
This paper presents an analysis of linear and circular consecutive k-out-of-n:G systems using the continuous-time Bayesian network (CTBN) approach. The main contributions include the development of CTBN models for these systems and the subsequent reliability analysis. Additionally, the paper derives exact expressions for system reliability and mean time to failure (MTTF). Numerical illustrations are presented. It is concluded that the reliability of circular system is more than linear system.

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Modern Iterative Optimization with Monotone Operators and Nonexpansive Mappings

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key-words: Halpern iteration, proximal gradient method, strong convergence.

Abstract:
In recent years, convex optimization and fixed point theory have emerged as indispensable tools for solving complex inverse problems in engineering and computer sciences. While the standard proximal gradient method is highly efficient for real-world tasks such as image deblurring and denoising, its application to high-dimensional problems is often hindered by slow convergence rates. To overcome this limitation, inertial extrapolation techniques are widely employed. However, a fundamental theoretical challenge arises in infinite-dimensional Hilbert spaces, where most existing accelerated algorithms rely heavily on the Mann-type iteration, guaranteeing only weak convergence.

In this study, we introduce a novel Halpern-type inertial proximal gradient algorithm that simultaneously achieves acceleration and strong convergence guarantees. We propose a new three-step hybrid iterative scheme that unifies the geometric properties of Halpern anchoring, inertial momentum, and proximal splitting operators. This structural approach elegantly resolves the theoretical paradox between the "shoot forward" nature of inertial extrapolation and the "pull backward" stabilization of Halpern anchoring.

Under very relaxed control conditions on the parameter sequences, we prove the strong convergence of the proposed scheme to an optimal solution of the minimization problem without assuming monotonicity. Furthermore, we extend our generalized optimization theorem to solve the prominent Split Feasibility Problem (SFP) in set intersections. Computational experiments via PSNR and SSIM metrics confirm that our novel algorithm significantly outperforms classical and weakly-convergent inertial algorithms in terms of both operational speed and structural edge preservation in digital image processing.

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A New Nonparametric k-Sample Test Based on Interval Counts of Order Statistics

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key-words: K-sample problem, nonparametric test, order statistics.

Abstract:
In this study, we propose a new nonparametric test statistic for the k-sample problem, which tests the null hypothesis that k independent samples are drawn from the same continuous distribution. The proposed test is an extension of the two-sample precedence-type tests and based on the placement of order statistics. Specifically, the test statistic is constructed by summing the counts of observations from one sample falling into intervals defined by the order statistics of another sample, considered symmetrically across all pairs of groups. Due to the complex dependency structure of the statistic, we employ a permutation-based approach to determine critical values. Monte Carlo simulations are conducted to evaluate the power of the proposed test and compare its performance against the traditional Kruskal-Wallis test under various location and scale shift scenarios. A numerical example is also provided to illustrate the applicability of the proposed test.

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Fixed Point Theorems for Nonlinear Contractive Mappings in Cone-Valued θ-Type Multiplicative Metric Spaces

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key-words: Banach algebra, contractions, metric.

Abstract:
Fixed point theory plays a fundamental role in nonlinear analysis and has significant applications in differential equations, integral equations, optimization, and applied mathematics. Inspired by recent developments in generalized metric structures, this paper introduces and studies Geraghty-type contractive conditions in the setting of cone θ-type multiplicative metric spaces defined on ordered Banach algebras. By extending the classical notions of θ-metric and multiplicative metric spaces, we construct a framework based on solid multiplicative cones and establish their essential topological and convergence properties. Within this generalized structure, we formulate new fixed point theorems for self-mappings satisfying cone-valued θ-type multiplicative Geraghty contractions, which weaken the traditional Lipschitz condition while preserving the existence and uniqueness of fixed points in complete spaces. The proposed results significantly extend and unify several well-known contraction principles, including those of Banach, Kannan, Chatterjea, and standard Geraghty contractions, under a broader cone-valued multiplicative setting. Furthermore, we develop extensions to higher-dimensional frameworks to enhance applicability in complex nonlinear systems. Illustrative examples are presented to substantiate the theoretical findings and to demonstrate the effectiveness of the introduced approach in generalized analytical environments.

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Dynamic Aspects of Composition Operators on Weighted Holomorphic Function Spaces

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key-words: Composition operators, mean ergodic, holomorphic functions.

Abstract:
Let X be a complex Banach space, and let B_X denote the open unit ball. We denote by $H(B_X)$ the space of all holomorphic function $f : B_X \to \mathbb{C}$. Any continuous and bounded mapping $v : B_X \to (0, \infty)$ is called a weight. For the weight v defined in this way, we define the following weighted Banach spaces of holomorphic functions on B_X

$$H_v(B_X) = \{f \in H(B_X) : \|f\|_v = \sup_{x \in B_X} v(x)|f(x)| < \infty\}.$$

We investigate the dynamical properties of composition operators acting on the weighted space of holomorphic functions $H_v(B_X)$. By using the asymptotic behavior of the inducing self-maps $\varphi : B_X \to B_X$, we obtain explicit sufficient criteria ensuring that the associated composition operator is power bounded or uniformly mean ergodic. Several examples are provided to illustrate the applicability of the obtained results. Many researchers have investigated the dynamic properties of composition operators on various spaces of holomorphic functions, and extensive studies on this topic can be found in the literature [1, 2, 3, 4].

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Simultaneous Approximation by Bivariate Kantorovich-Type Operators in Weighted Spaces

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key-words: Bivariate sampling operators, simultaneous approximation, weighted spaces.

Abstract:
This study introduces a theoretical framework for bivariate Kantorovich-type sampling operators in weighted function spaces. We investigate the rate of simultaneous approximation using multidimensional Taylor expansions and mixed moduli of smoothness. We demonstrate that the mathematical derivative of the operator smoothly approaches the generalized derivative of the function. Furthermore, we discuss the practical implications of this theoretical property in digital image processing. Since digital images are discrete structures that are inherently non-differentiable, classical edge detection methods often amplify noise. We present a methodological concept where the derivative of the continuous operator kernel serves as an analytical edge detector. This mathematical architecture proposes a deterministic, one-step method for simultaneous image denoising and feature extraction, bridging pure approximation theory with potential applications in medical imaging analysis.

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Risk-Aware Autonomous Decision Analysis under Cross-Domain Distribution Shift Using Uncertainty and Explanation Entropy

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key-words: Deep Ensemble, domain shift, epistemic uncertainty, focus entropy, explainable AI, risk-aware triage, diabetic retinopathy

Abstract:
Deep learning-based diabetic retinopathy (DR) screening may become unreliable under domain shift across acquisition settings and populations [4]. This study proposes an entropy- and uncertainty-based framework for identifying potentially erroneous autonomous decisions under cross-domain shift.

For each input x, let $\mathcal{M} = \{f_m\}_{m=1}^{M}$ denote an ensemble of M probabilistic classifiers. Ensemble outputs yield epistemic uncertainty, maximum confidence, and prediction margin [2]. Grad-CAM and SmoothGrad maps are generated, and focus entropy measures the dispersion of model attention.These signals are integrated into an XGBoost triage model that estimates whether an autonomous decision is likely to be incorrect.

The framework was evaluated using EyePACS, APTOS 2019, and Messidor-2, with IDRiD used for explanation-based validation. The base system comprised three EfficientNet-B0 classifiers [3]. A model trained on EyePACS [1] achieved 82.39% source-domain accuracy but 69.15% on Messidor-2, with recall decreasing to 40.72%. Incorrect predictions could remain overconfident, and explanation maps frequently shifted from retinal lesions toward less informative regions.

The risk filter achieved 94.75% AUC. Of 3,662 images, 2,485 (68%) were assigned to a low-risk autonomous group with no observed errors; the remaining 1,177 (32%) were referred for specialist review and contained all 512 base-model errors. Ensemble uncertainty, focus entropy, and risk triage therefore provide an interpretable reliability layer for autonomous screening under domain shift.

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Approximation by Generalized Baskakov Operators Associated with Exponential Functions

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key words: Baskakov operators, Stancu-type extension, exponential functions, uniform convergence, Voronovskaya-type asymptotic theorem.

Abstract:
This study introduces a novel Stancu-type extension for a specific class of Baskakov operators frameworks linked with exponential functions. Initially, the paper delves into the core geometric and algebraic properties of these newly constructed operators, providing precise computations for their moments. By employing exponential test functions, the uniform convergence of these operators within the Banach space $C^*[0,\infty)$ is rigorously demonstrated.

Furthermore, the manuscript evaluates the qualitative degree of approximation, establishing quantitative bounds using the framework of the exponential modulus of continuity. Lastly, a Voronovskaya-type asymptotic theorem is formulated and proved to characterize the first-order asymptotic expansion of the operators. The insights and theorems established in this research offer meaningful advancements to the field of positive linear operators, effectively generalizing previous literature on exponential-preserving Baskakov-type operators used for alignment for the different equations

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An Inertial Projection-Based Neurodynamic Approach for Image Restoration and Inpainting

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key-words: Inertial neurodynamic model, projection-based RNN, image inpainting, image restoration, lyapunov stability.

Abstract:
The ℓ_1-regularized least squares optimization, widely known as the LASSO problem, is a fundamental mathematical framework for sparse signal recovery, image restoration, and image inpainting. While recurrent neural networks (RNNs) provide an effective real-time parallel computing environment for such tasks, existing neurodynamic models often face limitations due to high computational overhead in dimension-doubling variable splitting or slow convergence in first-order architectures. To address these challenges, this paper proposes a novel Inertial Projection-Based Recurrent Neural Network (IP-RNN). By integrating a Prox-Projection identity to manage the non-differentiable ℓ_1 subgradient within the original problem space and incorporating a second-order inertial extrapolation term to accelerate the convergence rate, the proposed model achieves significant computational efficiency. Through a rigorous theoretical analysis using Lyapunov stability theory, we prove the global asymptotic stability of the system, ensuring its convergence to the optimal solution. Numerical experiments on various image restoration and inpainting tasks demonstrate that the proposed IP-RNN consistently achieves competitive results in terms of Signal-to-Noise Ratio (SNR) and Structural Similarity (SSIM) indices while maintaining a more compact and faster architecture compared to conventional neurodynamic approaches.

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Weinstein Pseudo-Differential Operators on Torus

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key-words: Pseudo-differential operators, Weinstein transform, discrete Weinstein transform.

Abstract:
Pseudo-differential operators are combinations of integral transform and partial differential operators on $\mathbb{T}^{n+1}$. This theory plays a significant role in various problems involving partial differential operators by exploiting the discrete Weinstein transform on $\mathbb{Z}^{n+1}$. In this talk, the concept and various properties of the Weinstein pseudo-differential operators will be discussed in relation to the discrete Weinstein transform.

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The Difference of Squares Formula and Quadratic Equations

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key-words: Difference of squares, quadratic equations, alternative solving methods.

Abstract:
The history of solving quadratic equations stretches back over four thousand years to Ancient Babylon, where scribes calculated land areas and astronomical cycles using rules remarkably similar to our own. However, most cuneiform texts provide only "recipes"—step-by-step instructions without explaining the underlying logic. Paradoxically, modern mathematics education often mirrors this ancient approach, favoring universal formulas (the "standard recipe") while neglecting more elegant and efficient alternatives. This study demonstrates that a wide range of problems typically solved via standard quadratic equations can be addressed more effectively through the direct application of the difference of squares formula.

We propose a specific algorithm based on this approach, highlighting its relevance for mathematical modeling. Many real-world problems, when initially translated into mathematical language, naturally take a form that is ideally suited for the difference of squares method before they are forcibly reduced to a standard quadratic form. By revisiting techniques once championed by Diophantus, Al-Khwarizmi, and Fibonacci—methods largely overshadowed since the 17th century—this work offers a streamlined alternative to the "general recipe," enhancing both computational efficiency and conceptual clarity in the classroom.

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Multi-State Performance Analysis of Benes Network

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key-words: Multi-state Benes network, reliability, availability.

Abstract:
The multi-state Benes network is a widely used interconnection architecture in communication and parallel processing systems due to its fault-tolerant and rearrangeable non-blocking characteristics. This paper presents a comprehensive reliability and performance analysis of a multi-state Benes network using a combined Markov Universal Generating Function (CMUGF) approach. The proposed methodology integrates continuous-time Markov processes with the UGF technique to effectively model the stochastic behavior and multiple performance states of switching elements and links under failure and repair mechanisms. A generalized CMUGF algorithm is developed to evaluate important system performance measures, such as reliability, availability, mean time to failure (MTTF), sensitivity analysis and Birnbaum importance measures. The proposed algorithm successfully computes all targeted performance measures for the constant failure and repair rates while efficiently handling the large state-space complexity associated with multi-state systems. Numerical results are compared with existing research work to validate the accuracy and effectiveness of the proposed methodology. The comparative analysis demonstrates improved computational efficiency and enhanced reliability assessment capability of the developed framework.

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Convergence of the Generalised Picard-Mann Iteration for G-Monotone Non-Expansive Mappings in Banach Spaces with a Graph

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Key-words: Generalised Picard–Mann iteration, G-monotone nonexpansive mappings, uniformly convex Banach space.

Abstract:
In this paper, we study the convergence of the generalised Picard–Mann iteration for approximating fixed points of G-monotone nonexpansive mappings in uniformly convex Banach spaces endowed with a directed graph structure. A mapping $T : C \to C$ on a nonempty closed convex subset C of a Banach space X is called G-monotone nonexpansive if it preserves the edge relation of a directed graph $G = (V(G), E(G))$ with $V(G) = C$ and satisfies $\|Tx - Ty\| \leq \|x - y\|$ for all $(x, y) \in E(G)$. The iterative scheme under investigation is defined by $x_{n+1} = T^k((1-\alpha_n)x_n + \alpha_n Tx_n)$, where $k \geq 1$ is a fixed positive integer and $\{\alpha_n\} \subset (0, 1)$ is a control sequence. We first observe that the iteration developed by Wen (2019) is recovered as a special case when $k = 2$, thereby establishing that the generalised Picard–Mann scheme is strictly more general. We then prove two edge propagation lemmas which rigorously verify that the graph structure, specifically the membership of the iterates and their images in the edge set $E(G)$, is preserved throughout the iteration under the assumptions that G is connected and transitive and that G-intervals are convex. These lemmas are essential for the valid application of the G-monotone nonexpansive condition at each step and address a gap that is often treated only informally in the existing literature. Using these results, we establish three main convergence theorems. First, we prove a necessary and sufficient condition: $F(T) \neq \emptyset$ with $(x_0, p) \in E(G)$ for some $p \in F(T)$ if and only if the iterative sequence $\{x_n\}$ is bounded and satisfies $\lim_{n\to\infty} \|x_n - Tx_n\| = 0$, provided $\lim_{n\to\infty} \alpha_n(1 - \alpha_n) > 0$. Second, we prove weak convergence of $\{x_n\}$ to a fixed point under Opial's property and Property (P). Third, we establish strong convergence when the mapping is additionally semicompact, including a proof that $F(T)$ is closed. Finally, we present a numerical example and demonstrate through comparative computations that the generalised Picard–Mann iteration converges significantly faster than the Wen iteration to the fixed point.

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A New Gradient-Based Neurodynamic Network for Circular Cone Constrained Variational Inequality Problems

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key-words: Circular cones, neurodynamic networks, variational inequalities.

Abstract:
In this study, we propose a new gradient-based neurodynamic network for solving variational inequality problems constrained by circular cones. Circular cones generalize second-order cones by allowing adjustable aperture parameters, thereby providing greater modeling flexibility in constrained optimization problems. By reformulating the circular cone constrained variational inequality problem into an equivalent system of smoothed equations, we design a steepest-descent neurodynamic system whose equilibrium points correspond exactly to the solutions of the original problem. Using Lyapunov stability theory, we establish the global stability and convergence of the proposed network. Numerical experiments are presented to demonstrate the effectiveness and feasibility of the proposed approach.

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A Projection Recurrent Neural Network for Circular Cone Constrained Variational Inequalities

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key-words: Circular cone, variational inequality, projection recurrent neural network.

Abstract:
In this paper, we develop a projection recurrent neural network for solving nonlinear variational inequality problems constrained by circular cones, together with related optimization models. Since the circular cone extends the second-order cone, it offers a more general framework for the formulation and analysis of constrained problems. By employing a smoothing technique, the original variational inequality problem is reformulated as an equivalent circular cone programming problem. Based on the relationship between optimal solutions of circular cone programming and the associated Karush–Kuhn–Tucker conditions, we construct a projection-based recurrent neural network with reduced dimensionality and lower computational complexity. Moreover, Lyapunov stability theory is applied to show that the proposed neural network is Lyapunov stable and globally convergent to a solution satisfying the reduced optimality conditions. Numerical examples are included to demonstrate the effectiveness and practical applicability of the proposed approach.

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The Numerical Range of the Generalized Inverse and Its Applications to Iterative Methods

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key-words: Numerical range, generalized inverse, EP operators, steepest descent.

Abstract:
The numerical range (field of values) of a bounded linear operator is a convex set in the complex plane that contains the spectrum—a property known as spectral inclusion. For singular or ill-conditioned operators, however, the standard spectral inclusion can be exceedingly loose, and traditional inverse theory fails. This talk investigates how the *generalized inverse*, defined variationally as the unique least squares solution of minimal norm [2], can be used to sharpen spectral inclusion and to analyse iterative methods for singular linear equations.

In this talk, we establish a geometric characterization of the closed range condition (which is an essential hypothesis for a bounded generalized inverse) via a gap at zero in the numerical range of the associated positive operator $\bar{T} = T^*T|_{\mathcal{R}(T^*)}$. Secondly, for EP operators (those satisfying $TT^\dagger = T^\dagger T$), we prove a sharpened spectral inclusion:

$$\sigma(T) \subseteq \overline{W(T)} \cap \frac{1}{\overline{W(T^\dagger)}},$$

and we characterize EP operators geometrically by the reciprocity of the numerical ranges of T and $T^\dagger$ on the reducing subspace $\mathcal{R}(T^*)$.

Finally, we show that the convergence of the Petryshyn, Schulz, steepest descent, and conjugate gradient methods for computing $T^\dagger b$, is governed entirely by the numerical range of $\bar{T}$. The convergence factors for these methods, including the optimal parameter for the Petryshyn iteration and the error reduction factor for steepest descent are expressed directly in terms of the bounds

$$m_W = \inf \mathrm{Re}(W(\bar{T})) = \|T^\dagger\|^{-2}, \text{ and } M_W = \sup \mathrm{Re}(W(\bar{T})) = \|T\|^2.$$

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A New Approach to Decision Making Using Soft Covering Based Rough Sets by Concept of Soft Maximal Description

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key-words: Soft rough set, soft covering based rough set, decision making.

Abstract:
Soft covering based rough set theory has become a useful and well-known area of research in theories of uncertainty. The present work follows up on this flourishing research topic. While soft covering based rough sets defined via the concept of soft minimal description have been extensively used by various authors in the literature for decision making problems, soft covering based rough sets defined via the concept of soft maximal description have not yet been applied in this context. Motivated by this gap, the present study applies these models within different decision making frameworks. In all cases, the best alternative is selected from a finite set, and in some methods, a ranking of alternatives is also produced. Also, the obtained results are compared in order to evaluate the effectiveness of the considered decision making approaches.

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Hyperbolic Pythagorean Hodograph Curves

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key-words: Pythagorean hodograph curve, Poincaré upper half–plane, Poincaré metric.

Abstract:
In this paper we introduce and completely classify hyperbolic Pythagorean hodograph (H–PH) curves in the Poincaré upper half–plane. Unlike the Euclidean case, where polynomial PH curves form a rich closed class, we show that rational functions constitute the natural and maximal class in the hyperbolic setting. We prove a necessary and sufficient condition for the existence of rational H–PH curves and derive a canonical normal form. Besides, we demonstrate the existence of unit speed rational H-PH curves as a property distinct from the Euclidean case. A precise comparison with Euclidean PH curves is also established.

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4th International Conference: Constructive Mathematical Analysis

Characterization of Some Classes of Compact Operators on Cesàro Difference Series Spaces

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key-words: Cesàro means, difference operator, series spaces, compact operators.

Abstract:
In this study, it is aimed to compute the norms and the Hausdorff measures of noncompactness of matrix operators acting on the Cesàro difference series space $|C_\alpha|_p(\nabla)$, which is generated by means of the difference operator and the Cesàro mean [1]. Furthermore, by applying the Hausdorff measure of noncompactness, we characterize certain classes of compact operators on Cesàro difference series space. Finally, by making use of these results, some known classical results in the literature are generalized.

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4th International Conference: Constructive Mathematical Analysis

Modules with ss-Supplements in Their Cofinite Extensions

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key-words: Semisimple modules, ss-supplement submodules, cofinite extensions, semiperfect rings, CE_{ss}-supplementing modules.

Abstract:
The concept of CE_{ss}-supplementing modules is introduced and analyzed. A module T is termed CE_{ss}-supplementing whenever each cofinite extension of T contains an ss-supplement of T. It is demonstrated that for modules over commutative von Neumann regular rings, the properties of cofinite injectivity, being CE_{ss}-supplementing, and possessing property (CE) coincide. Consequently, for left V-rings, it follows that a module is CE_{ss}-supplementing precisely when it has property (CE). The property CE_{ss}-supplementing transfers to direct summands. The amply CE_{ss}-supplementing property of a module is equivalent to the requirement that every submodule inherits the CE_{ss}-supplementing condition. It is proved that a ring S is semiperfect with $Rad({}_SS) \leq Soc({}_SS)$ if and only if every S-module is (amply) CE_{ss}-supplementing.

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